Analytical Modeling of Solar Cells
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Abstract: In the present work, a modified method is utilized to find the real roots of nonlinear equations of a single-diode PV cell by combining the modified Aitken's extrapolation method (MAEM), Aitken's extrapolation method (AEM) and the Newton-Raphson method (NRM), describing, and comparing them. The extrapolation method (MAEM) and (AEM) in the form of Aitken Δ²–acceleration is applied for improvement the convergence of the iterative method (Newton-Raphson) technique. Using a new improve to Aitken technique on (NRM) enables one to obtain efficiently the numerical solution of the single-diode solar cell nonlinear equation. The speed of the proposed method is compared with two other methods by means of various values of load resistance (R) in the range between R ∈ [1, 5] and with the given voltage of the cell V_{pv} as an initial value in ambient temperature. The results showed that the proposed method (MAEM) is faster than the other methods (AEM and NRM).

Keywords: Modified Aitken's method; Aitken's method; Newton-Raphson method; PV cell; single-diode design

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1. Introduction

Photovoltaics, known as PV cells, convert solar radiation directly into electricity and are manufactured using solar panels covered with crystalline or non-crystalline elements, the most important of which is silicon. There are many types of silicon solar cells: A monocrystalline silicon cell in its industry, the efficiency of this type is 16%. A polycrystalline silicon solar cell with 13 % efficiency. Amorphous cells: In which layers of silicon are deposited on solar panels, the efficiency of this type is (3×6) %, and less expensive than the previous two types[1-15]. The world of spacecraft's and satellites alone is full of solar applications. Scientists are pushed to manufacture a new type of solar cell that is not in the same effective traditional sources, but much cheaper, more useful, and widely used. Known that solar cells or photovoltaic cells turn light into electricity and run many devices from "computers" to computers Satellites and solar cells "voltaic images", it is an electronic cell generated electric driving force exposed to light radiation[16-23]. Thin film solar cells is a solar cell made of several layers of chips that work by the light effect to convert solar energy into electrical energy. The thickness of the layers varies between several nanometers to tens of microns. So far, combinations of the following elements have proven their usefulness in exploiting the effect of a voltage light to produce an electric current from the falling sunlight. These elements follow group I (alkaline elements), group III (terrestrial alkaline elements), and group VI, according to the periodic table of elements[24-40]. Numerical methods are emerging in mathematical research and they have a wide range in
many applications in engineering and sciences such as optimal control problems, integral equations, and fractional differential equations, etc\cite{[41-58]}. The goal of the present work is to describe a new method in order to find the real roots of single-diode nonlinear equation of the solar cells. It is organized as follows: section 2 characterizing the analytical model of a single-diode design of the photovoltaic cell; Section 3 establishing the root finding NRM, AEM and MAEM; section 4 results and discussion; section 5 conclusions of the obtained results. All the results are obtained using Matlab 2019.

2. Characteristics of Single-Diode Solar Cells Equation

The simple equivalent electric circuit of a PV cell shown in Figure 1.

![Equivalent-circuit of single-diode model.](image)

Using Kiechhoff's current law for the current I, the equation of this equivalent circuit is given by

\[
I = I_{ph} - I_D \tag{1}
\]

\[
I_D = I_0 \left(\frac{V_{pv}}{e^{V_{pv}/V_T}} - 1 \right) \tag{2}
\]

\[
I = I_{ph} - I_0 \left(\frac{V_{pv}}{e^{V_{pv}/V_T}} - 1 \right) \tag{3}
\]

where:

- \(I_{ph}\) is the photocurrent (A);
- \(I_0\) is reverse saturation current of the diode (A); I and \(V_{pv}\) are the delivered current and voltage, respectively (V); \(V_T = \frac{kT}{q} = 0.0259\) V is thermic voltage = 27.5 \(\equiv\) 26 mV at (\(T = 25\)°C Air-Mass = 1.5); m is the recombination factor closeness to an ideal diode (1 \(<\) m \(<\) 1.5), \(k\) is Boltzmann constant = \(1.38 \times 10^{-23}\) J/K; T is p-n junction temperature (K); \(q\) is the electron charge= \(1.6 \times 10^{-19}\) C.

\[
I_{ph} = I_{so} \tag{4}
\]

\[
I_D = I_s \left(\frac{V_{pv}}{e^{V_{pv}/V_T}} - 1 \right) \tag{5}
\]

Merge Eq. 4 in Eq. 5 we get

\[
(I_{so}) - 10^{-12} \left(e^{V_{pv}/V_T} - 1 \right) = \frac{R}{V} \tag{6}
\]

where \(n\) ideally factor 1 \(<\) n \(<\) 2, \(I_s\) reverse saturation current= \(10^{-12}\) A. In parallel, \(V_D = V_{pv} = V\).

Eq. 6 can be applied to determine \(V\) of the cell mathematically with the first derivative of this equation.

3. Analysis of the Mathematical Methods

3.1 Newton-Raphson Method (NRM)

It is an effective algorithm to find real dependent roots. Therefore, it is an example of root finding algorithms. It can be used to find the upper and lower limits of such functions, by finding the roots of the first derivative of the function. The geometric interpretation is as follows: we choose a maximum value close to the "root of the equation". We change the graphical representation by tangent and calculating the approximate zero. Zero tangent is an approximate value of the root of the equation, and then can be recalculated to get a closer solution to the root. In practice: operations for \(f: [a, b] \rightarrow R\), a defined and derivative function on the field \([a, b]\) choose a nominal value of \(x_0\) (the closer it is to the solution, the better). Determine by reference for each natural integer \(n\)

\[
x_1 - x_0 = \frac{f(x_1) - f(x_0)}{f'(x_1)} \text{ then} \tag{7}
\]

\[
x_{n+1} = x_n + \frac{f(x_n)}{\hat{f}(x_n)} \tag{8}
\]

where \(\hat{f}(x_n)\) is the derived function of the \(f(x_n)\) function.

We can show that if \(\hat{f}(x_n)\) is a continuous function and the unknown root \(\alpha\) is isolated, then there is an
adjacent to $\alpha$ where all the starting values of $x_0$ for the neighborhood, the successive $x_n$ approach the $\alpha$. Moreover, if $f'(\alpha) \neq 0$, the quadratic convergence, i.e., the number of integers is almost doubled at each stage.

This process is repeated until the convergence criterion is satisfied:

$$|x_i - x_{i-1}| < \varepsilon$$  \hspace{1cm} (10)

It is apparent that for every approximation $x_{i-1}$, a better one ($x_i$) of the actual solution $x_i$ can be achieved through Eq. 6, $x_i$ is at the intersection of the function’s tangent at $x_{i-1}$ and axis $x$.

3.2 Aitken’s Method (AEM)

In numerical analysis, the Aitken squared delta operation or Aitken Extrapolation is a sequential acceleration method, used to accelerate the sequence convergence rate. It is named after Alexander Aitken, who introduced this method in 1926.

In general, let the sequence $\{ \bar{E}_n \}$ can be described by

$$\bar{E}_n = E_{n+2} - \frac{(E_{n+2} - E_{n+1})^2}{E_{n+2} - 2 \times E_{n+1} + E_n} n = 0, 1, 2, ...$$  \hspace{1cm} (11)

for acceleration the convergence of Eq. 11 can be written as

$$\bar{E}_n = E_n - \frac{(E_{n+1} - E_n)^2}{E_{n+2} - 2 \times E_{n+1} + E_n} n = 0, 1, 2, ...$$  \hspace{1cm} (12)

3.3 Modified Aitken’s Method (MAEM)

For a given $x_0$

$$\bar{E}_0 = E_2 - \frac{(E_2 - E_1)^2}{E_2 - 2 \times E_1 + E_0}$$

$$\bar{E}_1 = E_3 - \frac{(E_3 - E_2)^2}{E_3 - 2 \times E_2 + E_1}$$

$$\bar{E}_2 = E_4 - \frac{(E_4 - E_3)^2}{E_4 - 2 \times E_3 + E_2}$$

Define the improve value of $\bar{E}$ using

$$\bar{E} = \bar{E} = E_2 - \frac{(E_2 - E_1)^2}{E_2 - 2 \times E_1 + E_0} \bar{E}_2$$

In general, $\{ \bar{E}_n \}$ can be defined by

$$\bar{E}_n = E_{n+2} - \frac{(E_{n+2} - E_{n+1})^2}{E_{n+2} - 2 \times E_{n+1} + E_n} n = 0, 1, 2, ...$$ \hspace{1cm} (13)

The above equation is the proposed method used to improve (AEM).

The procedure of NRM obtain in the following steps:

INPUT initial approximation solution $x_0 = 1$,

tolerance $\varepsilon = 10^{-9}$, maximum number of iterations $N$, $f$, $df$.

OUTPUT approximate solution $x_{n+1}$

**Step 1:** Set $i = 1$

**Step 2:** Calculate $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for $n = 0, 1, 2, ...$

**Step 3:** If $|x_i - x_{i-1}| < \varepsilon$, then go to Step 6

**Step 4:** Set $x_0 = x$

**Step 5:** $n = n + 1$, $i = i + 1$, go back to Step 2.

**Step 6:** OUTPUT $x_{n+1}$ and stop iteration.

The procedure of MAEM obtain in the following steps:

**Given:** $x_0$, $\varepsilon = 10^{-9}$, $N$, $f$, $df$

**Step 1:** For $i = 1$ to 2

**Step 2:** Calculate $\bar{E}_n = E_{n+2} - \frac{(E_{n+2} - E_{n+1})^2}{E_{n+2} - 2 \times E_{n+1} + E_n}$ for $n = 0, 1, 2, ...$

**Step 3:** If $f(x_i) = 0$ or $f(x_i) < \varepsilon$, then go to Step 6

**Step 4:** Set $E_{n+1} = \bar{E}_n$

**Step 5:** $n = n + 1$, $i = i + 1$, go back to Step 2.

**Step 6:** OUTPUT $x_{n+1}$ and stop iteration.

4. Results and Discussion

Consider the Eq. 1 is modeled in the form single-diode PV cell has obtained the following approximate solutions and the three different methods are applied with the initial value $x_0 = 1$. In Table 1 the methods NRM, AEM and MAEM with the comparison of the solution results is given and listed in the last column of this table when the load resistance $R = 1$.
Table 1. The $\epsilon$ for $V$ of PV cell with $R = 1$ by comparison with three different methods.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Vpv-NRM</th>
<th>Vpv-AEM</th>
<th>Vpv-MAEM</th>
<th>$\epsilon$-NRM</th>
<th>$\epsilon$-AEM</th>
<th>$\epsilon$-MAEM</th>
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<td>1</td>
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<td>0.947037857</td>
<td>0.923295275</td>
<td>0.028583139</td>
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<td>0.000000000</td>
<td>0.000000000</td>
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</table>

Figure 3. Obtained solutions of the study result.

The obtained solution plot in the (no of iteration- $\epsilon$)-plane proves that the proposed method (MAEM) have small iterations compared with the other method. Parallel to this feature, it is also noted that the proposed method (MAEM) has a behavior of the solution in the initial value $x_0 = 1$ has the smallest error tolerance compared with (NRM) and (AEM).

In Table 2 the methods NRM, AEM and MAEM with the comparison of the solution results is given and listed in the last column of this table when the load resistance $R = 2$. 

<table>
<thead>
<tr>
<th>Iterations</th>
<th>Vpv-NRM</th>
<th>Vpv-AEM</th>
<th>Vpv-MAEM</th>
<th>$\epsilon$-NRM</th>
<th>$\epsilon$-AEM</th>
<th>$\epsilon$-MAEM</th>
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</table>
Table 2. The $e$ for $V$ of PV cell with $R = 2$ by comparison with three different methods.

Figure 4 Presents the obtained solutions of the study result.

Figure 4. Obtained solutions of the study result.

In Table 3 the methods NRM, AEM and MAEM listed in the last column of this table when the load resistance $R = 3$.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$V_{pv}$-NRM</th>
<th>$V_{pv}$-AEM</th>
<th>$V_{pv}$-MAEM</th>
<th>$e$-NRM</th>
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Table 3. The $e$ for $V$ of PV cell with $R = 3$ by comparison with three different methods.

Figure 5 Presents the obtained solutions of the study result.

In Table 4 the methods NRM, AEM and MAEM with the comparison of the solution results is given and listed in the last column of this table when the load resistance $R = 4$.

<table>
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<th>Iterations</th>
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</table>

Table 4. The $e$ for $V$ of PV cell with $R = 4$ by comparison with three different methods.
Table 4. The $\varepsilon$ for $V$ of PV cell with $R = 4$ by comparison with three different methods.

Figure 6 Presents the obtained solutions of the study result.

In Table 5 the methods NRM, AEM and MAEM with the comparison of the solution results is given and listed in the last column of this table when the load resistance $R = 5$.

Table 5. The $\varepsilon$ for $V$ of PV cell with $R = 5$ by comparison with three different methods.

Figure 7 Presents the obtained solutions of the study result.
Results of tables 1 to 5 are showing that the suggested method (MAEM) are having low error after relatively view iterations are computed, and this in turn is demonstrating their efficiency.

5. Conclusion

In this paper, we give three numerical solutions for Mathematical model of the single-diode PV cells. The main advantage of the developed method is simplicity and high accurate approximate solution is achieved using a few numbers of iterations. The obtained numerical results are comparing with some other methods.

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