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Article

Scheme selection with probabilistic multiple objectives optimization

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Abstract: This article presents the scheme/alternative selections with probabilistic multiple objectives optimization (PMOO). In the PMOO assessment, all response objectives (attributes) are divided into beneficial or unbeneficial types according to their function and preference to equivalently contribute their partial preferable probabilities simultaneously, the total preferable probability of an alternative is the multiplication of its all partial preferable probabilities, which determines the optimal evaluation uniquely and comparatively. The application examples contain a personnel selection and a production quantity optimal control. The former is for an engineer position in a software development department from five alternative candidates withstanding seven optimal criteria (response objectives) comparatively, and the latter aims to get an optimal production quantity with a higher profit rate and lower final cost. In the personnel selection, the seven optimal response objectives (criteria) include relevant education, work experience in the field, relevant certificates, level of presentation and communication, ability of personnel management, capabilities of planning and organization, and expressiveness of foreign language. All these seven response objectives are attributed to the beneficial type of attribute to join the assessment. As to the production quantity control, the profit rate belongs to the beneficial type of objective, while the final cost is attributed to the unbeneficial type of attribute. The evaluated results reveal that the optimal alternate for the personnel selection is candidate No. 4, and the optimum production quantity is at $x^* = 54$ items. The achievement of the present article indicates the validity of the corresponding approach and algorithm with rationality. The novelty of this work is to reflect the simultaneity of the response objectives (criteria) in the optimal system by using probabilistic multiple objectives optimization, and all response objectives either beneficial type or unbeneficial type are evaluated separately in an equivalent manner.

Keywords: scheme selection; probabilistic multiple objectives optimization; simultaneity of optimization; overall optimization; systems theory

1. Introduction

Selection of a scheme or alternative with multiple criteria or many attributes is an optimization problem with many objectives. Some authors define multi-criterion decision-making (MCDM) as the process of ranking or evaluating alternatives based on a set of confrontation criteria [1–3]. There have been various MCDM methods proposed [4], which include, SAW, ELECTRE, DEMATEL, AHP, TOPSIS, PROMETHEE, COPRAS, VIKOR, WASPAS, EDAS, CoCoSo, WISP, and ARAS, etc. [4].

However, the simultaneous optimization of many objectives in a system is a challenging problem, though the above "multi-criterion decision-making methods" were used to handle the relevant problems frequently, the fatal shortcoming of these methods is not aware. The intrinsic drawback of the above methods is the main lack of reflecting the simultaneity of many optimal objectives in the system, more details were stated in [5,6].

Recently, probabilistic multiple objectives optimization (PMOO) was proposed, which could reveal the simultaneity of many objectives in the optimal system properly [5,6]. In the PMOO, the optimal objectives (attributes) are fundamentally discriminated by either beneficial type or unbeneficial type, and then all attributes of both beneficial type and unbeneficial type are taken as independent events to contribute their partial preferable probabilities with equivalent regulations separately. The independence of events was discussed in [5,6]. Furthermore, the total (overall, integral) preferable probability of each alternative is obtained by the product of its all partial preferable probabilities. Finally, the total (overall, integral) preferable probability of each alternative is reasonably taken as the unique representative to conduct the ranking of the alternatives in the optimization problem with multiple objectives. Undoubtedly, the fundamental viewpoint of systems theory and treatment of probability theory are employed in PMOO [5,6]. The drawbacks of the simultaneity of many optimal objectives in the previous methods in the optimized system have been overcome [5–8]. PMOO adopted the fundamental viewpoint of systems theory and treatment of probability theory to set up the overall optimization of a system.

In the present article, the scheme/alternative selections with probabilistic multiple objectives optimization (PMOO) are illuminated through examples.

2. Methods

The probabilistic multiple objectives optimization (PMOO) is briefly demonstrated here first [5,6].

In a multiple objectives optimization (MOO) problem, some attribute utility indexes have the features of "higher value being more welcomed" in general, i.e., the objective welcomes that with higher value, which gets more preference inevitably [5,6]. This type of objective is called a beneficial type of attribute index. In this case, a new term "preferable probability" was introduced to characterize the "preference degree" of the attribute in the optimization process for each candidate reasonably [5,6]. Furthermore, for simplicity, it assumes that the partial preferable probability of this type of attribute response is proportional to its specific utility value of the alternative positively, i.e.,

$$P_{ij} = A_j Y_{ij}, i = 1, 2, 3, \dots, \alpha; j = 1, 2, 3, \dots, \beta. \tag{1}$$

In Equation (1), α is the total number of alternatives in the system; β represents the total number of objective (attribute) indicators of each alternative; Y_{ij} is the value of utility value of the j-th objective (attribute) indicator of the i-th alternative; P_{ij} reflects the partial preferable probability of the beneficial type of objective Y_{ij}; A_j indicates the scaled factor of the j-th beneficial type of objective.

In accordance with the general principle in probability theory, for the j-th objective (attribute) index, the following expression for the scaled factor A_j is obtained [5,6,9].

$$A_j = 1/(\alpha \overline{Y_j}) \tag{2}$$

In Equation (2), $\overline{Y_j}$ is the averaged value of utility of the j-th objective indexes in the attribute group.

On the other hand, some attribute utility indexes have the features of "the lower value being more welcomed" in a MOO problem, i.e., the objective welcomes that with lower value inevitably, which gets more preference [5,6]. In this case, the partial preferable probability of the unbeneficial objective is linearly related to its attribute utility index value negatively in the optimization process equivalently,

$$P_{ij} = B_j(Y_{jmax} + Y_{jmin} - Y_{ij}), i = 1, 2, 3, \ldots, \alpha; j = 1, 2, 3, \ldots, \beta. \tag{3}$$

In Equation (3), both Y_{jmin} and Y_{jmax} express the minimum and maximum values of the utility of the objective indicators in the j-th attribute group, respectively; B_j is the scaled factor of the j-th unbeneficial type of objective index, which can be expressed as [5,6],

$$B_j = 1/[\alpha(Y_{jmax} + Y_{jmin} - \overline{Y_j}) \tag{4}$$

As to the continuous (non discrete) utility functions $Y_j(x_1, x_2, \ldots, x_v)$ of the criterion indicators in the j-th attribute of the beneficial type of objective, its partial preferable probability is written as [5],

$$P_{Y_j} = Y_j(x_1, x_2, \ldots, x_v)/[\int_{\Omega} Y_j(x_1, x_2, \ldots, x_v) dx_1 dx_2 \ldots dx_v] \tag{5}$$

In Equation (5), v is the number of the variables, Ω indicates the integral domain of the v variables.

Correspondingly, for the continuous (non discrete) utility functions $Y_j(x_1, x_2, \ldots, x_v)$ of the criterion indicators in the j-th attribute of the unbeneficial type of objective, its partial preferable probability is written as [5],

$$P_{Y_j} = [Y_{jmax} + Y_{jmin} - Y_j(x_1, x_2, \ldots, x_v)]/\{\int_{\Omega}[Y_{jmax} + Y_{jmin} - Y_j(x_1, x_2, \ldots, x_v)] dx_1 dx_2 \ldots dx_v]\} \tag{6}$$

Again, by the general principle, in probability theory [9], the total (overall, integral) preferable probability of the i-th alternative could be written as [5,6],

$$P_i = P_{i1} \cdot P_{i2} \cdot \ldots \cdot P_{ij} \cdot \ldots = \prod_{j=1}^{\beta} P_{ij} \tag{7}$$

Moreover, if a weighting factor w_j for j-th attribute response is involved, the total (overall) preferable probability of the i-th alternative could be formulated as follows [5,6],

$$P_i = P_{i1}{}^{w_1} \cdot P_{i2}{}^{w_2} \cdot \ldots \cdot P_{ij}{}^{w_j} \cdot \ldots = \prod_{j=1}^{\beta} P_{ij}{}^{w_j} \tag{8}$$

Distinctly, in this assessment, the total (overall) preferable probability of each alternative is the unique/decisive index in the optimization process. This is the fundamental characteristic of probabilistic multiple objectives optimization (PMOO). Besides, through these procedures using Equations (1)–(8), the optimization problem with multiple objectives now becomes an optimization issue of a single objective one in terms of total (overall) preferable probability naturally. Thus the total (overall) preferable probability of each alternative can be used to perform the ranking of the alternatives in the optimization problem with multiple objectives. The optimum alternative corresponds to the candidate that gets the highest total (overall) preferable probability, which is therefore the optimized result of the corresponding optimization problem with multiple objectives.

Besides, the details of conflicts or interdependence between objectives were discussed by using cluster analysis in detail [5].

Many applications of the above approach are performed, which gave acceptable results and conformed to known, and indicated the reasonability of this approach [5,6].

In the next section, examples are given to illuminate the scheme/alternative selection processes with probabilistic multiple objectives optimization.

3. Results

In this section, two examples are given.

3.1. Example 1: A personnel selection for an engineer position in a software development department

The assessment of a personnel selection for position of an engineer in a software development department is raised and will be conducted by using PMOO approach. In this problem, it involves five alternative candidates (M_1 to M_5) and seven criteria: O_1—Relevant education, O_2—Work experience in the field, O_3—Relevant certificates, O_4—Level of presentation and communication, O_5—Ability of personnel management, O_6—Capabilities of planning and organization, and O_7—Expressiveness of foreign language.

The scores of all alternative candidates with each criterion are shown in **Table 1** [4]. In the assessment of this issue, all criteria of the attributes belong to a beneficial type of index, i.e., their utility value has the feature of "higher value being welcomed" in the evaluation.

Table 1. The scores of all alternatives.

Alternative	**Attribute**						
	O_1	O_2	O_3	O_4	O_5	O_6	O_7
M_1	4	3	3	3	4	3	4
M_2	4	3	4	4	3	4	4
M_3	5	5	4	3	3	4	3
M_4	4	5	3	4	4	5	3
M_5	5	4	3	3	4	4	3
Weighting factor	0.11	0.13	0.13	0.15	0.15	0.15	0.16

The assessment results with PMOO of this issue are presented in **Table 2**.

The evaluated results shown in **Table 2** reveal that the alternative candidate 4 gains the highest total (overall) preferable probability, and gets a position of rank 1 in the assessment with probabilistic multiple objectives optimization. Therefore, the optimized selection is the alternative 4. In **Table 2**, the symbol "P_{O1} *for* O_1" represents the partial preferable probability of objective (attribute) O_1.

As a comparison, **Table 3** cited the assessed results given by Stanujkic with MULTIMOORA, WISP and WASPAS [4].

The ranking orders in **Tables 2** and **3** are different overall though there are some rankings that are the same by coincidence, which indicates the intrinsic difference between PMOO and other methods. PMOO could embody the simultaneity and replacement of the response objectives (attributes, criteria) in the optimal system, which conforms to the essence of multi-objective optimization intrinsically.

Table 2. Assessment result of the alternative selection with PMOO.

Alternative	P_{O1} for O_1	P_{O2} for O_2	P_{O3} for O_3	P_{O4} for O_4	P_{O5} for O_5	P_{O6} for O_6	P_{O7} for O_7	Total (overall) preferable probability, P_t	Rank
M_1	0.1818	0.15	0.1765	0.1765	0.2222	0.15	0.2353	0.1899	5
M_2	0.1818	0.15	0.2353	0.2353	0.1667	0.2	0.2353	0.2058	3
M_3	0.2273	0.25	0.2353	0.1765	0.1667	0.2	0.1765	0.2061	2
M_4	0.1818	0.25	0.1765	0.2353	0.2222	0.25	0.1765	0.2184	1
M_5	0.2273	0.20	0.1765	0.1765	0.2222	0.20	0.1765	0.2014	4

Table 3. The ranking order of candidates given by Stanujkic with MULTIMOORA, WISP and WASPAS.

Alternative	MULTIMOORA	WISP	WASPAS
M_1	5	5	5
M_2	3	2	2
M_3	2	3	3
M_4	1	1	1
M_5	3	4	4

3.2. Example 2: A determination of production quantity for an electronic part

An electronic part is produced in one factory. A proper plan for production quantity is needed.

It is known that the profit rate f_1 of the production varies with the production quantity x in a function form of $f_1(x) = 10\cdot(6 - 0.01x)\cdot\sin(\pi x/250)$, and the final cost f_2 is a function of x as $f_2(x) = 2.5^2 - (2.5 - 0.01x)^2$; above functions are valid for the production quantity x in the range of $0 < x < 250$ items. In this issue, the problem is to find an optimal production quantity x^* that makes the maximum profit rate $f_1{}^*$ and minimum final cost $f_2{}^*$ simultaneously.

Obviously, the criterion of the profit rate $f_1(x)$ belongs to the beneficial type of index, while the criterion of the final cost $f_2(x)$ is attributed to the unbeneficial type of index in the evaluation in this issue. Therefore, the optimal problem is now written as,

$$\begin{cases} \text{Max}\, f_1(x) = 10 \cdot (6\text{-}0.01\text{x}) \cdot \sin(\pi\text{x}/250); \\ \text{Min}\, f_2(x) = 2.5^2 - (2.5\text{-}0.01\text{x})^2; \\ 0 < x < 250. \end{cases} \tag{9}$$

Since the profit rate $f_1(x)$ and the final cost $f_2(x)$ are continuous functions instead of discrete ones, so the Equations (5) and (6) could be employed rationally.

Thus the evaluation of the partial preferable probability P_{f1} of the profit rate $f_1(x)$ is,

$$P_{f_1}(x) = f_1(x)/[\int_0^{250} f_1(x)dx] \quad (10)$$

While the maximum value of the final cost $f_2(x)$ is $2.5^2 = 6.25$, and the minimum value is 0 in the range of $0 < x < 250$, respectively. Correspondingly, the evaluation of the partial preferable probability P_{f2} of the final cost f_2 is,

$$P_{f_2}(x) = [6.26 - f_2(x)]/\{\int_0^{250} [6.25 - f_2(x)]dx\} \quad (11)$$

Furthermore, the total preferable probability $P_t(x)$ for production quantity x is,

$$P_t(x) = P_{f_1}(x) \times P_{f_2}(x) \quad (12)$$

Obviously, the position of $P_t(x)$ taking maximum value is the same as that of the function $G(x) = 10\cdot(6 - 0.01x)\cdot\sin(\pi x/250)\cdot (2.5 - 0.01x)^2$.

As a result, it derives the solution of function $G(x)$ getting its maximum value at $x^* = 54$ items. Therefore, the optimum solution of this issue is $x^* = 54$ items, the corresponding profit rate $f_1{}^*$ is 34.2719, and the final cost $f_2{}^*$ is 2.4084.

4. Conclusion

The probabilistic multiple objectives optimization is an effective methodology to address the scheme or alternative selection. In the PMOO assessment, the optimal response objectives (attributes) are fundamentally discriminated into either beneficial type or unbeneficial type, and then all objectives whether the beneficial type or unbeneficial type are evaluated to obtain their partial preferable probability with equivalent regulations separately, in which the fundamental viewpoint of systems theory and treatment of probability theory are employed. The achievement of the present article reveals the validity of the corresponding approach and algorithm. The potential future research directions of this approach might include applications in many fields, such as industry, transportation finance, etc., and appropriate combination with numerical analysis and computation as well.

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