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Article

Research on bridge safety early warning method based on strain energy theory and health monitoring data

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Abstract: Bridges are technology-intensive and heavily invested permanent infrastructure. After completion and opening to traffic, bridge structures are easy to be affected by factors such as traffic load and atmospheric environment. Therefore, it is necessary to do safety warning and evaluation of bridges, especially the abnormal behavior in the early stages of bridge operation. In this research paper, a large-span continuous rigid frame bridge installed with the health monitoring system (HMS), of which a large amount of health monitoring data are collected by the HMS, is used as an example, and then a bridge safety early warning method is proposed when the bridge is during early operating period. First of all, the research finding that the internal stress of the prototype bridge obeys normal distribution through statistical analysis is used; next, we deduce that the strain energy inside the prototype bridge is subject to the Non-central Chi-square distribution combined with the strain energy and statistical theories; in the end, the key probability density distribution function of strain energy and its parameters are derived by using the key stress distribution function of the high performance concrete C50 strength grade used in the prototype bridge. The method recommended in this paper is conducive to the formulation of bridge preventive maintenance strategies.

Keywords: large-span continuous rigid frame bridge; health monitoring system; bridge safety early warning; punctiform strain energy; non-central chi-square distribution

1. Introduction

The bridge is in a random atmospheric environment in long time. Due to the influence of materials aging, stress redistribution because of shrinkage and creep, environmental corrosion, fatigue effects and other factors, the concrete strength, structure and component resistance of the reinforced concrete structures will decay with varying degrees [1], or some structural abnormalities happens. Bridge accidents happen from time to time and cause great loss to people's life and property safety. Once the bridge is damaged, the consequences are extremely serious. The bridge accidents have made people realize the importance of health monitoring system for the main purpose of safety during the operation of the bridge. Some newest techniques are recommended to apply in innovative sensing systems for the health monitoring of civil structures, such as optical fiber [2,3]. Nowadays, the technology of health monitoring is widely used in long-span bridge structures, and many progresses have been archived in the area of SHM of Large-span structures, but there still exist many challenges, such as: Data-driven science and technologies, identify damage accurately and quantitatively etc. [3,4].

At present, many kinds of analysis and assessment method of mass monitoring data generated from the SHM of Large-span bridge structure are proposed by experts and scholars [5–9]. However, there are few reports on the application of strain energy theory to the analysis and processing of health monitoring data of the civil structures, especially in the early stages of bridge operation, safety warnings for abnormal structural behavior. Cornwell [10] proposed a method to detect and locate damage for beam-like structures based on the changes in the strain energy of the structure. Liang et al. [11] presents an improved modal strain energy correlation method（MSEC）method for the identify the damage of truss bridge structures, where the prediction of modal strain energy change vector is differently obtained by running the eigen solutions on-line in optimisation iterations. Hu et al. [12] presents a damage detection of surface crack in composite laminate, of which using mode shapes to compute the strain energy and define a damage index. Hu et al. [13] proposed a computationally attractive damage index method for structural damage detection of cylindrical shell. Liu and Zong [14] suggested a damage identification method with combining wavelet transform and strain energy to identify the structure damage of in-service transmission tower. Nobahari and Seyedpoor [15] put forward a method to identify multiple damage cases in structural systems using the concepts of flexibility matrix and strain energy of a structure. An efficient method is proposed by Seyedpoor and Yazdanpanah [16] to find multiple damage locations in structural systems, of which using the change of static strain energy (SSE) due to damage to establish an indicator for determining the damage location. Moradipour et al. [17] examined the application of an improved two-stage Modal Strain Energy (MSE) method with numerically and experimentally verified to a benchmark bridge available, taking a steel truss bridge as example. Alavinezhad et al. [18] adopt a common SHM method and modal strain energy that uses the changes in the dynamic properties of the structure for identifying the damage location and severity of Catwalk (access bridge) of an offshore oil and gas complex. Alavinezhad et al. [19] provide a connection tool between different offshore complex structures, and use modal strain energy for early damage identification in these structures. Alavinezhad et al. [20] suggested a novel damage index to increase the accuracy of damage localization by revising the modal strain energy-based Stubbs index for Foroozan offshore platform damage identification installed with structural health monitoring.

Nevertheless, the current research on various types of civil structures safety evaluation still lacks data, especially long-term health monitoring data. With using a large amount of strain monitoring data, combined with strain energy and statistical theories, for the first time, we present a method for the prototype bridge safety early warning, and it is very helpful for bridge engineers to develop prevent maintenance strategies and daily prevent maintenance of bridge structures.

2. Introduction of the background bridge with HMS

2.1. The health monitoring system (HMS) installed on the bridge

The sample bridge adopted in the paper is located in the Pearl River Delta region in Guangdong Province. The structure of the main beam of the bridge is a

(51.4 + 94 + 4 × 144 + 87) m prestressed concrete continuous box-beam system, with 8 main piers and 7 main spans. The box girder is a single box and single room section, and the full width of the box girder is 12.5 m, and the width of the bottom plate is 6.8m. The transverse slope of the bridge floor is 2.0%, and the longitudinal slope of the bridge floor is 0.15%. The whole bridge has 6 closing sections, and the heights of the main beam cross sections change from 8 m to 2.8 m according to 1.6 order power parabola from the supporting base to the mid-span. The thickness of the main beam base plate varies from 1 m to 0.32 m and thickness of the main beam web plate varies 0.9 m to 0.45 m. The prestressed tendons are $15\Phi^j 15.24$ mm steel strand with the strength: $R_y^b = 1860$ MPa, $2\Phi^j 12.7$ mm steel strand with the strength: $R_y^b = 1395$ MPa and high strength rebar respectively.

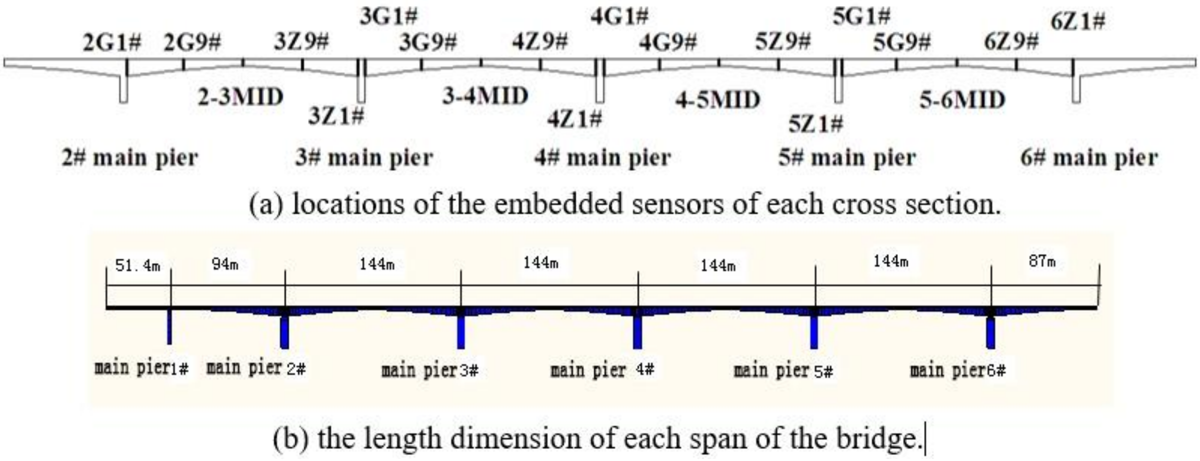


Figure 1. The diagrammatic sketch of cross section locations of the embedded sensors of HMS and the length dimension of each span.

The measuring points of the health monitoring system (HMS) in the main beam are arranged at the root of the cantilever, L/2 span, L/4 span and other key positions. There are 8 cantilever end sections, 8 L/4 sections and 4 L/2 sections respectively, as shown in **Figure 1** (**Figure 1a** illustrates the locations of the embedded sensors of each cross section; **Figure 1b** shows the length dimension of each span of the bridge.), and the embedded positions of sensors on each section are shown in **Figure 2**. The string type strain gauge (JMZX-215) and the centralized data acquisition module used in the bridge HMS are show in **Figure 3** (**Figure 3a** is the sample of the strain gauge; **Figure 3b** is the physical picture of the centralized data acquisition.), which are capable of simultaneously monitoring temperature values and having temperature compensation function. The sensor measuring time interval is 1 hour. The strain gauge parameters are shown in **Table 1** and the basic parameters of high performance concrete used in the bridge can be seen in **Table 2** [21,22]. The monitoring system of the prototype bridge mainly includes four parts: sensor system, data acquisition system, data management system, and evaluation and decision-making system. At present, and a large amount of strain monitored data have been obtained with the HMS.

Table 1. Parameters of the strain gauge.

Name	**Range**	**Sensitivity**	**Gauge length**	**Remarks**
Intelligent digital vibrating strain gauge	±1500 με	1 με	157 mm	Strain gauge embedded in concrete

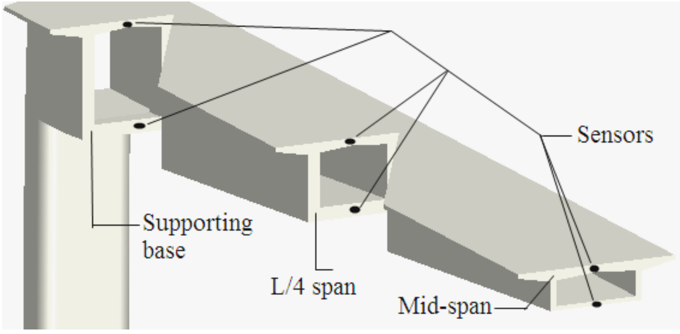


Figure 2. Position of the embedded sensors in half-span of the prototype bridge.

(a) **(b)**

Figure 3. Pictures of the JMZX-215 strain gauge and the centralized data acquisition module. **(a)** the JMZX-215 strain gauge; **(b)** the centralized data acquisition module.

Table 2. The basic parameters of high performance concrete used in the bridge.

Parameters	**Young's modulus (units: MPa)**	**Tensile Strength (units: MPa)**	**Compressive Strength (units: MPa)**	**Tensile failure strain (units: με)**
Value	3.45×10^4	3.278	55.12	0.95

2.2. The monitoring data processing

2.2.1. The profile of the pre-processed data

In the paper, the data acquired from the sensors named 2-3MID-1, 2-3MID-2, 5-6MID-2, 4Z9h-1 and 4G1h-1 are taken as examples to display the profile of the monitoring data, and the chosen time range is from March 2006 to April 2010. The description of pre-processing steps for HMS monitoring data can be seen in the papers [8,9], specifically, as follows:

(1) Read the sensor initial setting value after the casted concrete is solidified. As the sensors ware embedded before the concrete casting, the concrete hydration heat will produce initial strain in sensors. So, this value should be subtracted from the monitored strain value, of which the goal is to get setting values of the each sensor after the concrete is solidified.

(2) Subtract the shrinkage and creep strain values from the monitored strain value. As for those lacking of monitored data，they can use the finite element technology and build the finite element model of the bridge calibrated by field measured data to get the shrinkage and creep values. In this paper, we acquire the shrinkage and creep values corresponding to each sensor position from the long-term monitored strain data and then subtract this value from the measured strain values. The shrinkage and creep strain data extraction method is instructed in section 3.4 in detail. Of course, the extracted shrinkage and creep values are just approximate value.

(3) Subtract the thermal expansion strain value from the monitored strain value. As for the variation of environmental temperature, the monitored strain data include thermal strain. We mentioned on the above paper, the sensor adopted in this paper can simultaneously monitor temperature. So, we can easily remove the thermal strain from the monitored strain, the elimination formula is as follows:

$$\varepsilon_r = \varepsilon - (T - T_0)(F - F_0) \quad (1)$$

In the above formula: $F_0 = 10\mu\varepsilon/°\text{C}$， which is the coefficient of linear expansion of the sensor steel wire as for concrete bridges; T_0 is the initial temperature; F is the coefficient of linear expansion of structure; ε is the measured strain; T is the measured temperature.

After processing, the stress data can be converted from the processed strain data by the following formula:

$$\sigma = E \times \varepsilon \quad (2)$$

In the formula: E is the concrete elastic modulus, and the value is: $E = 3.45 \times 10^4$ MPa (28 days of age). This paper focuses on the performance of the bridge during early service, and the time lasts not long. During this time section, the change of modulus E tends to be stable，and so this paper neglect the effects caused by the concrete elastic modulus E.

Figure 4 shows the profile of the original data after several pre-processed steps. **Figure 5** shows the profile of the strain data after several pre-processed steps. The data interval in the figure below is mainly due to the loss of monitoring data caused by system failure.

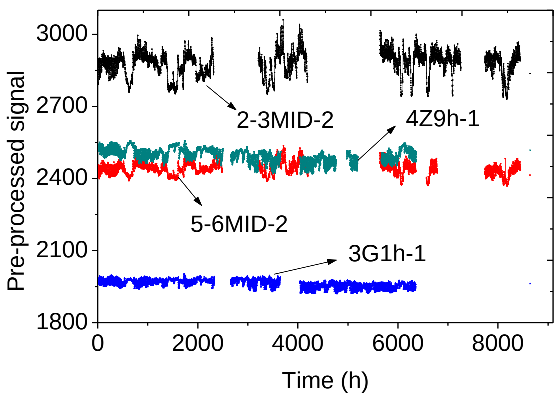


Figure 4. The profile of pre-processed data collected from the HMS.

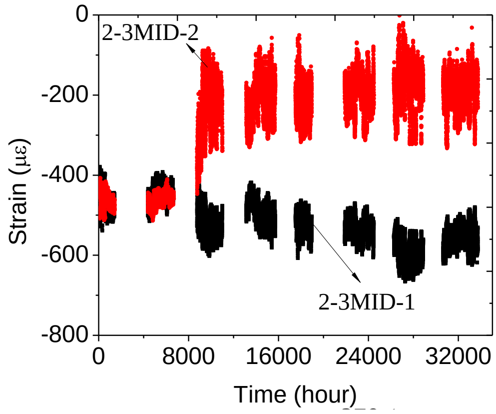


Figure 5. The profile of strain data transformed from the monitored data.

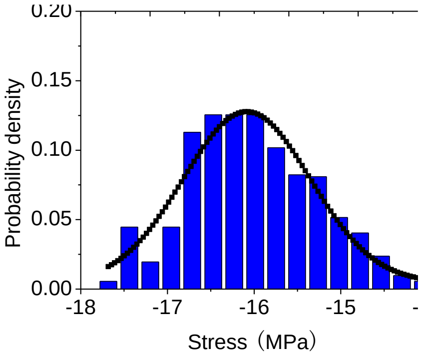
Seen from **Figures 4** and **5**, it can be found that sensors using electrical signals are susceptible to interference from external factors such as lightning. Therefore, preprocessing the initial signal is necessary. Also, the HMS is prone to malfunctions, resulting in the loss of some collected data.

2.2.2. The statistical distribution of stress transformed from the strain monitoring data

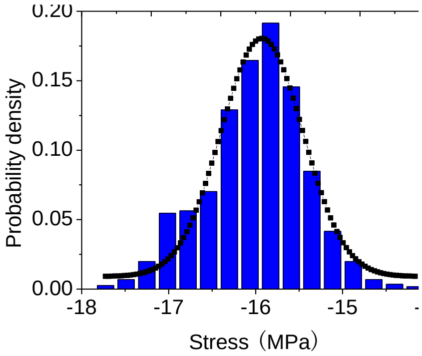
As for large-span prestressed concrete bridge structures, due to the creep role of concrete and relaxation of prestressed steel strand, the permanent load effect keeps changes with time in service. During the entire operation of the bridge, live load, permanent load, or structural resistance, has shown a great deal of uncertainty. During the design basis, the ability of bridge structures to complete the intended function is unpredictable in the specific operating environment. Therefore, the load statistics is very useful, which can help bridge managers to learn bridge work stress state and its trends in the future time.

Assuming that the main beam stress state change is in the stage of approximately linear elasticity during in early service, as the monitored strain data has been preprocessed, it can be transferred to stress by the Equation (2). Also, it is well known that the stress data obtained by the sensors in this article includes temperature-induced stress, vehicle load-induced stress, stress redistribution, etc.

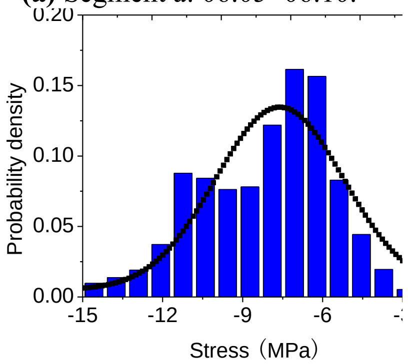
In reference [8], Li et al. have confirmed that the internal stress states of concrete bridges obey the normal distribution. In this report, the data collected from the sensor numbered 2-3MID-2 located in the mid-span web plate between the main pier 2 # and the main pier 3 # (Positions of the embedded sensors seen in **Figures 1** and **2**) is selected as example to learn the strain energy distribution, of which the main reason is that the stress measured in mid-span horizontal direction is close to the principal stress value. The stress statistics distribution and Gaussian distribution fitting of the transformed stress data are shown in **Figure 6**:



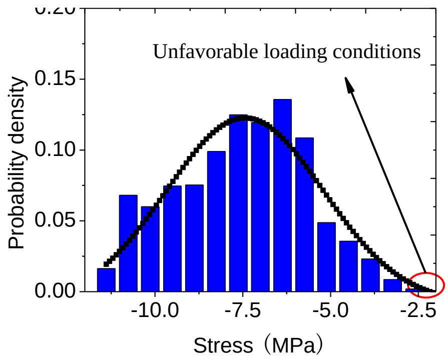
(a) Segment a: 06.05~06.10.



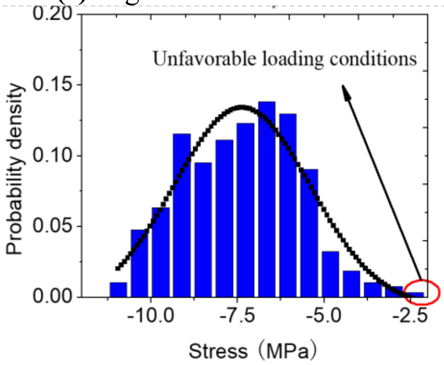
(b) Segment b: 06.11~07.04.



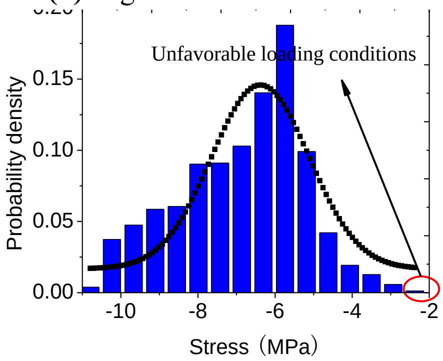
(c) Segment c: 07.05~07.10.



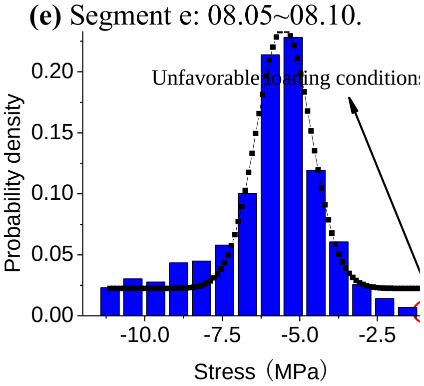
(d) Segment d: 07.11~08.04.



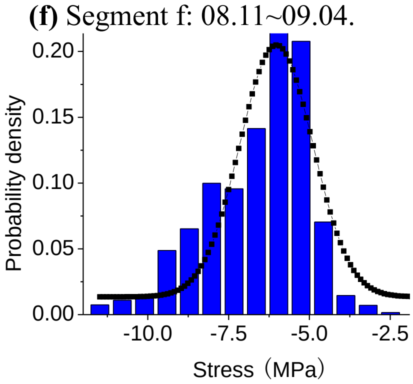
(e) Segment e: 08.05~08.10.



(f) Segment f: 08.11~09.04.



(g) Segment g: 09.05~09.10.



(h) Segment h: 09.11~2010.04.

Figure 6. Stress distribution statistics and Gaussian distribution fitting. **(a)** Segment a: 06.05~06.10; **(b)** Segment b: 06.11~07.04; **(c)** Segment c: 07.05~07.10; **(d)** Segment d: 07.11~08.04; **(e)** Segment e: 08.05~08.10; **(f)** Segment f: 08.11~09.04; **(g)** Segment g: 09.05~09.10; **(h)** Segment h: 09.11~2010.04.

Seen in **Figure 6**, the stress state of the mid-span base plate is gradually changed from compression to tension. Through data fitting analysis, we got the stress mean and variance of each statistical time period, seen in **Table 3**. Similarly, it can be seen from **Table 3** that the dispersion of monitoring data is relatively large.

Table 3. The stress means and variance of each statistical time period acquired from the sensor 2-3mid-2.

Time segment	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**
Mean	−17.89	−17.73	−9.42	−9.24	−9.18	−8.18	−7.32	−7.82
Variance	5.214	4.261	13.447	14.059	15.181	8.574	13.428	11.479

3. Main idea of strain energy calculation and statistics

3.1. The fundamental theory of strain energy

The potential energy stored in a body in the form of strain and stress is also called deformation energy, the strain energy formula is written as follows:

$$U = V\int_0^{\varepsilon_L} \sigma \mathrm{d}\varepsilon \tag{3}$$

On above: U is the strain energy of a body; V is the volume of a body; σ is the stress of the body; ε is the strain of the body. If the object is a linear elastic material, Equation (3) can be written as:

$$U = \frac{\sigma_l^2 V}{2E} \tag{4}$$

In the formula: E is the elastic modulus; V is the volume of a body; σ_l is the stress of the body.

3.2. Introduction to non-central Chi-square distribution

The non central chi-square distribution is a generalized form of chi-square distribution [23]. If $X_i, i = 1, \dots, k$ are k number of independent normally distributed random variable with a mean μ_i and a variance of σ_i^2, expressed as $N(\mu_i, \sigma_i^2)$, then the random variable $X = \sum_{i=1}^{k} (\frac{X_i}{\sigma_i})^2$ obey non-central chi-square distribution, and it's probability density function (PDF) can be expressed as:

$$f(x, k, \lambda) = \sum_{i=0}^{\infty} (\frac{e^{-\lambda/2}(\lambda/2)^i}{i!}) f(x; k + 2i) \tag{5}$$

Among: $\lambda = \sum_{i=1}^{k} (\frac{u_i}{\sigma_i})^2$; $f(x; v)$ is the probability density function representing a central Chi-square distribution of degree of freedom v, which can be expressed as follows:

$$f(x; v) = \frac{e^{-x/2} x^{v/2-1}}{2^{v/2} \Gamma(v/2)}, x \geq 0 \tag{6}$$

In the formula: $\Gamma(a)$ is Gamma function, and the expression is $\Gamma(a) =$

$\int_0^\infty t^{a-1}\, e^{-t} \mathrm{d}t$.

Mathematical expectation calculation formula is:

$$E(Y) = n\sigma^2 + \lambda \tag{7}$$

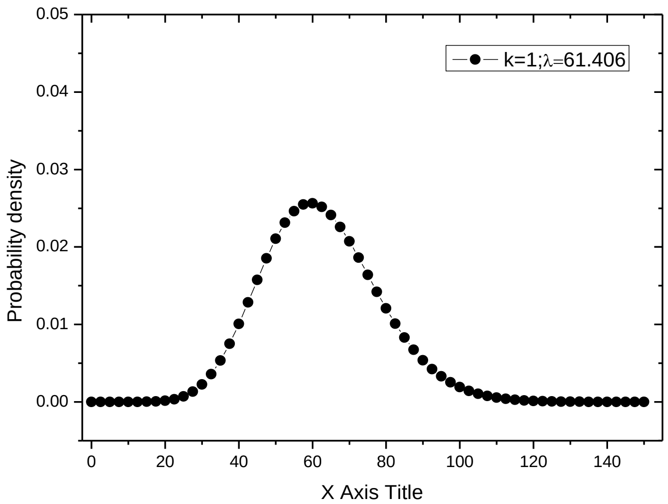
The variance calculation formula can be written as:

$$Var(Y) = 2n\sigma^4 + 4\sigma^2\lambda \tag{8}$$

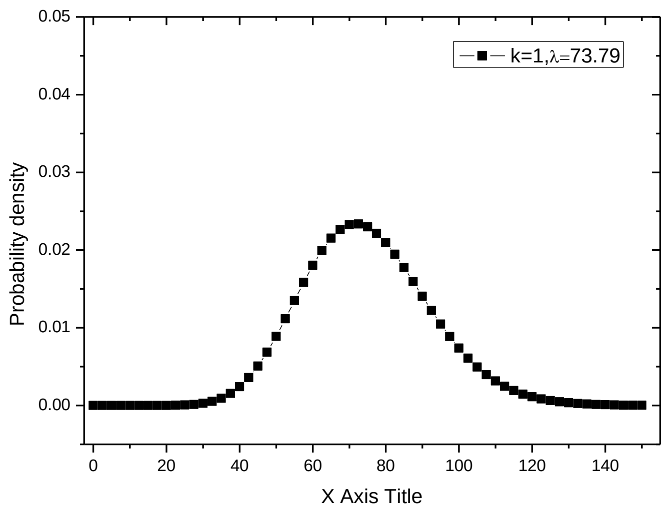
3.3. The statistical distribution of strain energy

Because the sensor named 2-3mid-2 is located in the middle floor of the bridge main span, the stresses in other directions can be ignored and so we think that the horizontal longitudinal direction stress transformed from the data acquired by the senor is approximately the principal stress at this position. In addition, the calculated strain energy in the paper only reflects the local strain energy of the sensor position, so we call it the punctiform strain energy.

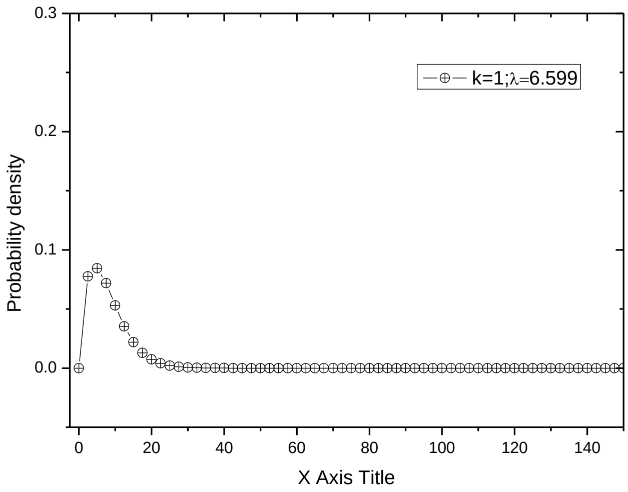
On the basis of statistical analysis result of **Figure 6** and Equation (4), supposing the volume V in Equation (4) take the value 1, we can draw a conclusion that the strain energy of per unit volume over a period of time follows a non-central chi-square distribution with the degree of freedom $\nu = 1$. Using the stress data in **Figure 6** and the calculation formulas in section 3.1 and 3.2, we can obtain the strain energy probability density function of non-central Chi-square distribution for each statistical time period corresponding to **Figure 6**, as shown in **Figure 7**:



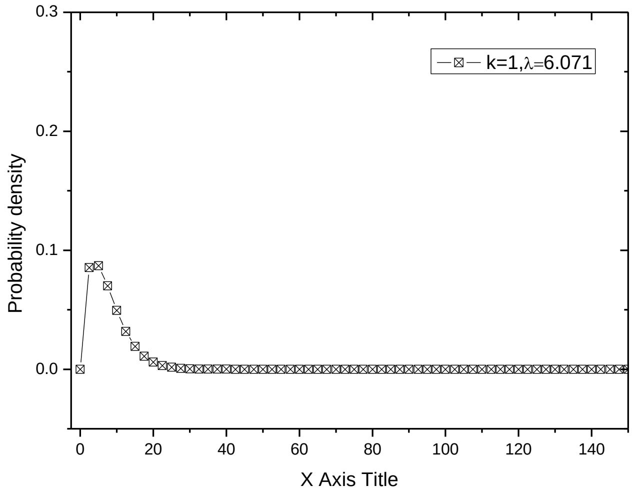
(a) Segment a: 06.05~06.10.



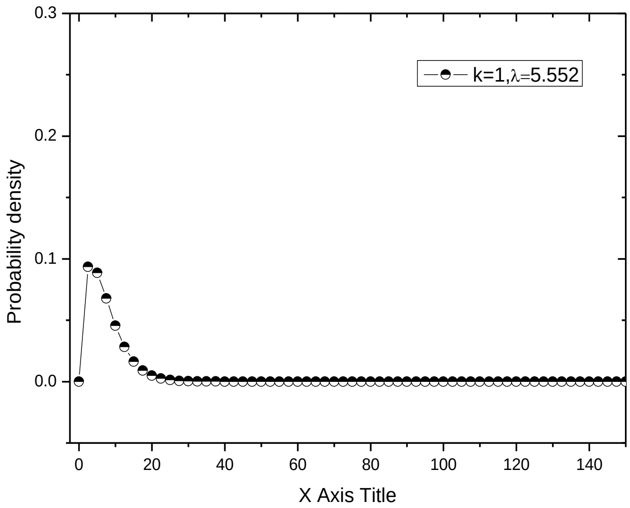
(b) Segment b: 06.11~07.04.



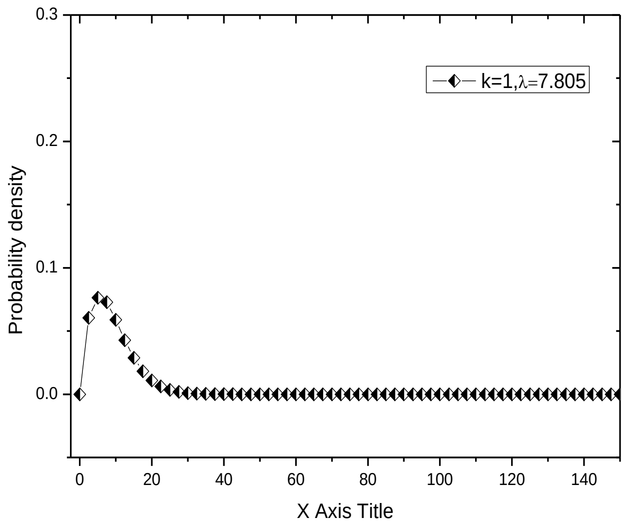
(c) Segment c: 07.05~07.10.



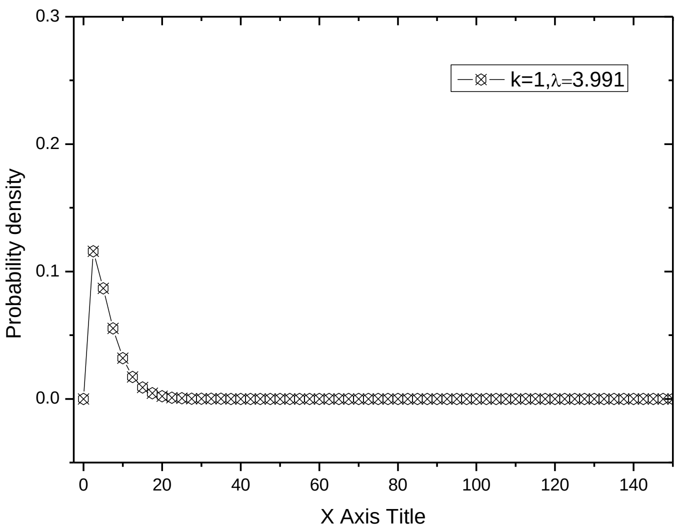
(d) Segment d: 07.11~08.04.



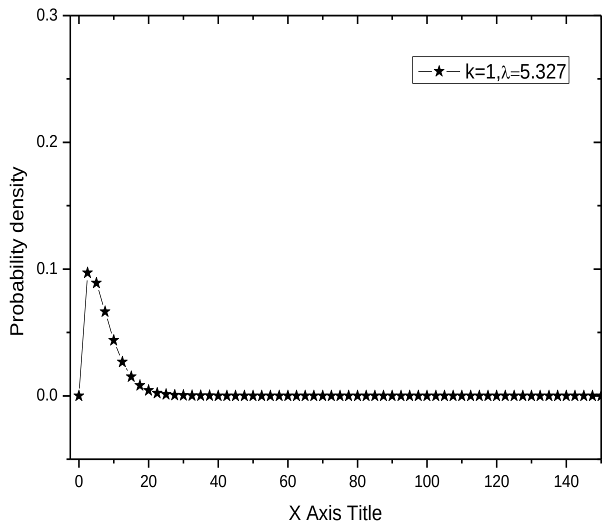
(e) Segment e: 08.05~08.10.



(f) Segment f: 08.11~09.04.



(g) Segment g: 09.05~09.10.



(h) Segment h: 09.11~2010.04.

Figure 7. The strain energy probability density function of non-central Chi-square distribution for each statistical time period. **(a)** Segment a: 06.05~06.10; **(b)** Segment b: 06.11~07.04; **(c)** Segment c: 07.05~07.10; **(d)** Segment d: 07.11~08.04; **(e)** Segment e: 08.05~08.10; **(f)** Segment f: 08.11~09.04; **(g)** Segment g: 09.05~09.10; **(h)** Segment h: 09.11~2010.04.

Through statistical analysis by non-central Chi-square distribution for each stress statistical time period, we can get the relevant statistical parameters, seen in **Table 4**.

Table 4. The statistical parameters of non-central Chi-square distribution of each stress statistical time period acquired from the sensor 2-3mid-2.

Time segment	a	b	c	d	e	f	g	h
λ	61.41	73.790	6.599	6.071	5.552	7.805	3.991	5.327
$E(Y)$	66.62	78.05	20.05	20.13	20.73	16.38	17.42	16.81
$Var(Y)$	1335.1	1293.99	716.59	736.74	798.06	414.70	575.00	508.15

4. Results and discussion

In order to help engineers do preventive maintenance for eliminating the early abnormal behavior warning of bridges effectively, in this article, the main research is on the definition of preventive maintenance indicators for the bridge early safety warning.

4.1. The key stress probability density distribution function

In order to obtain the key energy probability density distribution function parameter values, refer to the research findings of document [9], by using "Three Sigma" principle and the transformed stress data, combined with the minimum −2 MPa of pressure safety reserve requirement in the web plate, the key stress probability density distribution function is determined, and the diagram is as follows (see **Figure 8**):

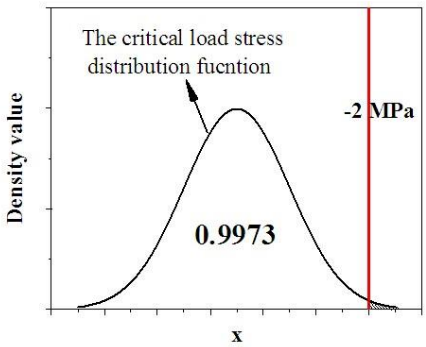


Figure 8. Diagram of the key stress probability density distribution function.

Based on **Figure 8**, we can get the standard deviation $\sigma_{th} = 1.447$ and the mean $\mu_{th} = -5.86$ MPa of the key stress probability density distribution function.

4.2. The key strain energy distribution function and conservation resource allocation

For helping bridge engineers rationally formulating bridge structure prevent maintenance strategies, especially for the prototype bridge safety early warning, in the paper, we define the critical probability density function of strain energy with non-central chi-square distribution.

Using the parameter values of the key stress probability density distribution function in Section 4.1, combining the Equations (5), (7) and (8); we can calculate the parameter values of the key strain energy non-central distribution function. The detailed parameters are as follows: $k = 1$, $\lambda_{th} = 16.401$, $E(Y)_{th} = 18.4948$, $Var(Y)_{th} = 141.7463$, and the diagram is as follows, seen in **Figure 9**:

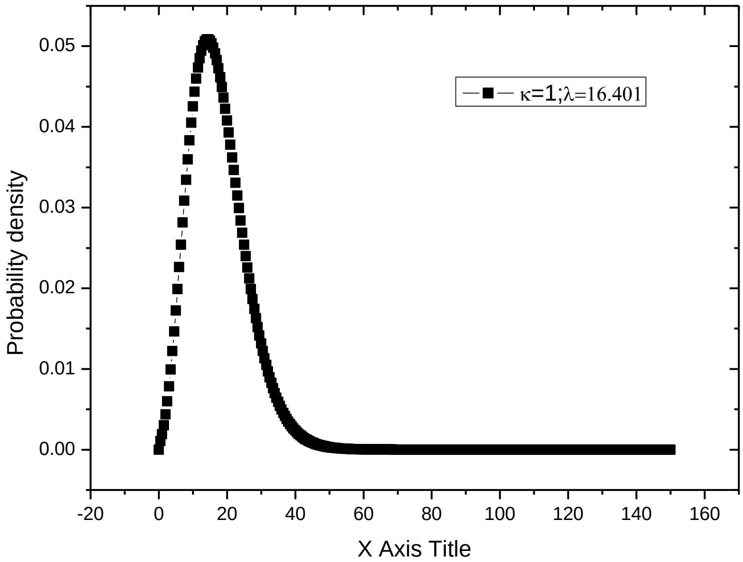


Figure 9. The profile of the key strain energy non-central Chi-square distribution function.

Throughout the above analysis, the pressure safety reserve of mid-span bottom plate between the main 2# pier and the main 3# pier is relatively low, and the values of non-central chi-square distribution parameter change greatly when the prototype bridge is in operation for about 1 year, of which the main reason may be the failure of load transfer of the prestressed steel strands. Therefore, the authors of this paper suggest that the bridge management engineers should firstly carry out preventive maintenance on the mid-span between the main pier 2# and the main pier 3#.

Of course, from the perspective of long-term operation of the prototype bridge, due to the factors such as material aging, the key thresholds determined above need to be appropriately adjusted.

5. Conclusions

In order to help bridge engineers timely detect abnormal behavior in bridge structures and develop effective bridge preventive maintenance strategies, especially in the early stages of bridge operation, using large amount of monitored strain data acquired from the HMS of the bridge, combining strain energy theory and Non-central Chi-square distribution, we suggest a bridge early safety warning method for determining the preventive maintenance of the bridge in the article, and the conclusions are as follows:

1) Based on the research finding that the internal stress of the bridge obeys the normal distribution, combining strain energy theory and Non-central Chi-square distribution, we found that the internal strain energy of the concrete material used in

the bridge follows non-central chi-square distribution. Doing statistical analysis of strain energy in different time periods of bridge operation, we obtain the variation trend of parameter values of non-central chi-square distribution of strain energy.

2) In order to achieve the goal of timely safety warning of bridge early structural abnormalities, in the report, we define the key probability density function of non-central chi-square distribution of strain energy, and the critical parameter values are: $k = 1,\ \lambda_{th} = 16.401,\ E(Y)_{th} = 18.4948,\ Var(Y)_{th} = 141.7463$.

3) In the next step research plan, we should focus on the bridge health monitoring system integrated with bridge maintenance strategy together, using strain energy method etc.

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