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Brief Introduction of the Application of Calculus Concepts to Geometry

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Abstract: Calculus has a long development history and its ideas can be traced back to the ancient Greek period. After the second crisis in mathematics, calculus thought keeps evolving and eventually formed the thought of calculus today. The idea of calculus is of great significance to mathematics and has a wide range of applications in geometric figures, such as calculating the quality of space geometry above geometry area.This paper briefly describes the development history of calculus, and analyzes the application of calculus ideas in geometry.
Keywords: Differential; Integral; Calculus Thought; Geometry

1. Research background

The importance of calculus to mathematics is self-evident. Taking buildings as an analogy, calculus to mathematics is as steel to architecture. Calculus has also made an indelible contribution to the development of human civilization. Calculus includes differential calculus and integral calculus, which studies functions from the part and the whole separately. Differential calculus mainly studies the local characteristics of functions, such as rate of change, extreme value, etc. The content of differential calculus is divided into derivative and differentia two main parts. Integral studies the total effect of accumulation of small changes as a whole. The process of taking the derivative is the differential method essentially. The process of finding the integral is the integral method essentially, which constitutes the main content of integral calculus with the nature, calculation, promotion and application of integral.

When it comes to calculus, people may naturally think of Newton and Leibniz, who are the founders of calculus. Their greatest contribution is that they summed up a series of rules for derivation and integration, indicating derivation and integration is an inverse pair, and deduced the famous Newton Leibnitz formula to solve this reciprocal inversion. Based on all these findings, differentiation and integration, which were originally developed independently, found a way to integrate and thus formed a mathematic subject, called calculus.

However, because the early calculus theory was not complete, it became the fuse of the second mathematical crisis. British bishop Berkeley questioned that Δx in the function was regarded as both 0 and not 0 during the derivation process, and so called it as the *Dead Ghost*. Therefore, mathematical voids, exemplified by Berkeley's paradox, have gradually emerged, waiting to be filled by another group of outstanding mathematicians. In the end, after unremitting efforts of countless outstanding scientists such as Cauchy, Abel, Will Strath, etc., the foundation of calculus was finally solidified. Calculus begins to show its unique mathematical charm.

The generation of calculus is mainly divided into three stages: the first stage is the emergency of the concept of limit; the second stage is the emergency of quadrature method; the last is the discovery of the reciprocal relationship between differentiation and integration. Although the idea of calculus can be traced back to the ancient Greek period, it was really developed until the second half of the 16th century. During this period, its concepts and related laws were produced and developed on the basis of Kepler and Cavalieri's method of indivisibles.

2. A brief introduction to calculus concepts

2.1 Derivative

There are two cases, one is the case where the derivative exists, and the derivative is defined as follow: Assume that *y=f(x)* is defined in the domain near the point x_0. When the independent variable x obtains an increment Δx, and x+Δx is still in the domain of x_0, the corresponding function increment is $\Delta y=f(x_0+\Delta x)-f(x_0)$. When Δx approaches 0 and $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ exists, then the function *y=f(x)* can be derivable at the point x_0 which function expression can be described into $f^{'}(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ or $f^{'}(x_0)=\frac{dy}{dx}=\frac{df}{dx}$. If the function *f(x)* is derivable in an open interval *I*, it means that for $\forall x\in I$, there is a unique *f'(x)* corresponding to it, so *f'(x)* is also a function, and *f(x)* is called the original function of *f'(x)* well *f'(x)* is the derivative function of *f(x)*.

The other case is where the derivative does not exist. Because the essence of the derivative is actually the limit and according to the definition of limit, If the function *f(x)* is meaningful in the deleted neighborhood of x, and for ∀ε>0 there is δ>0, and when $0<|x-x_0|<\delta$ there is $|f(x)-a|<\varepsilon$. Then a is called the limit of the function *f(x)* when x→x_0, marked as $\lim_{x\to x_0} f(x)=a$.

If the function is not defined at x_0, the necessary and sufficient condition for the function *f(x)* to be derivable at point x_0 is that the function exists at the left and right limits of x_0, and there are $\lim_{x\to {x_0}^+} f(x)=\lim_{x\to {x_0}^-} f(x)$.

2.2 Differential

The core idea of differentiation is infinite approximation. From a geometrical point of view, it means to divide a curve into several consecutive points, and then conduct subsequent calculations. Suppose Δx is the increment on the abscissa of point M on the curve y = f(x), Δy is the increment of the curve at point M corresponding to Δx on the ordinate, and dy is the increment of Δx on the ordinate corresponding to the tangent of the curve at point M. When |Δx| is small, |dy-Δy| is much smaller than |Δx| (high-order infinitesimal), so near the point M, we can approximate the curve segment with a tangent segment, which is described as the method of differentiation.

2.3 Integral

The concept of integral mainly comes from the process of finding the area, volume and arc length of some irregular figures. In fact, there is no specific concept of integral at the beginning. The method used is exhaustion which is similar to the current calculus, then followed by the more familiar cutting circle technique. By this method the circumference and area of a circle can be found out. Developed to the present, integral can be said to be the sum of continuous phenomena in essence, and its intuitive meaning is that it gives the total amount of the function in some region; it is a continuous simulation of the summation. For example, a single integral, that is, a definite $\int_a^b f(x)\,dx$, from a geometric point of view, the function is first differentiated from the interval [a, b], and then integrated and summed. It is equivalent to the cumulative sum of countless small rectangles essentially. Double integral $\int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x,y)dxdy$ actually means differentiating x and y, then accumulate and sum up each small cuboid, etc. Integral summation is now mostly used in the calculation of geometric figures.

3. The Application of Calculus Concepts to Geometry

3.1 Definite Integral

Definite integral is mainly used to find the area of plane figures. According to the definition of definite integral and its geometric meaning, the definite integral of function *f* on the interval [a, b] is equal to the algebraic sum of the areas of the figure enclosed by the function and the straight line of x=a, x=b as well as x-axis.

3.1.1 Find the area of a plane figure

For example, find the area of the graph enclosed by the function $y = e^x$and the line of x=0， x=2 as well as x-axis.

$$S = \int_0^2 e^x dx = e^x|_0^2 = e^2 - 1$$

3.1.2 Find arc length

The formula for arc length is$ds = \sqrt{(dx)^2 + (dy)^2}$， thus$S = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx = \int_a^b \sqrt{1 + (y')^2}\, dx$.

Meanwhile, if there is a parameter function $\begin{cases} x = \varphi(\theta) \\ y = \phi(\theta) \end{cases} (\alpha < \theta < \beta)$, and $x_\theta^{'}$、$y_\theta^{'}$ are continuous in the interval, then the length formula of the curve changes is $S = \int_\alpha^\beta \sqrt{(\varphi'(\theta))^2 + (\phi'(\theta))^2} dx$, And according to the calculation of dx, we can finally get $S = \int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$

3.2 Multivariate Differentiation

From a geometrical analysis point of view, if an area R on a plane is given, and the area of R is required to be calculated, at this time, if you use unary integral to calculate, you will find that it is not easy, because the geometric meaning of unary integral means the area enclosed by the curve and the x-axis. While what we really need is to calculate the area enclosed by the closed curve. This is where binary integrals are useful, and this is multivariable calculus.

3.2 Re-integration

3.2.1 Double Integral

The geometric meaning of the double integral is the directed volume of the curved top cylinder, and in the physical sense it can be said to be the pressure applied to the plane area. It is generally used rectangular coordinates to calculate the double integral. Double Integral in Cartesian Coordinates, $dV = dxdydz$, if you want to convert a double integral into a quadratic integral, one of the main points is to determine the upper and lower limits of the two definite integrals. While in a cylindrical coordinate system, $dV = \rho d\rho d\theta dz$, and in a cylindrical coordinate system, $dV = r^2 sin\varphi dr d\varphi d\theta$, it works.

According to the median value theorem of double integrals, if the function $f(x,y)$ is continuous over a bounded closed region D, and σ is the area, then there is at least one point (ξ,η) on D, which led to $\iint_D f(x,y) d\sigma = f(\xi,\eta) \cdot \sigma$.

Another application of double integral is to find the average value of the quantity in the area, that is to find the average value of the function f in the area R, which is generally used to find the position of the center of mass of the object.

Example 1: According to the geometrical meaning of double integral, calculate $\iint_D \sqrt{R^2 - x^2 - y^2} dxdy$, in which $D = \{(x,y) | x^2 + y^2 \leq R^2\}$.

Answer：By analyzing this problem, it can be concluded that the geometric meaning of the original problem is to find the volume of the working sphere with the center of the sphere at the origin and the radius of R, so the result is $\frac{2}{3}\pi R^3$.

3.2.2 Triple Integral

The geometric and physical meanings of triple integral are considered to be the mass of inhomogeneous space objects. It can be understood that the density of each point is different, so the density integration is performed for each point in the graph.

Triple integration is also a process of division, approximation, summation, and taking the limit. This process is compressed into one step, which is the triple integration operation.

$$\lim_{\lambda \to 0} \sum_{i=1}^{n} f(\xi_i, \eta_i, \varsigma_i) \Delta V_i \triangleq \iiint_{\Omega} f(x,y,z) dV$$

When calculating the triple integral, it is generally converted into a method of single integral plus double integral or double integral plus single integral, then the double integral is converted into cumulative integral, and finally the integral calculation is performed.

The triple integral and algorithm generally have the following two parts:

（1）Projection Method

First, the main thing is to distinguish from which direction the projection calculation will be less. Make a vertical line down from a point z', intersecting the figure at $z_1 = (x_0,y_0), z_2 = (x_0,y_0)$, Assume the lower half-surface of Ω in the z-axis direction is $z = z_1(x,y), z = z_2(x,y)$, Denote the projection of Ω on the xOy plane as D_{xy}.

As shown in the figure, first accumulate vertical dotted line segments, that is the density function (integrand) accumulates from a fixed value $z_1 = (x_0,y_0)$ to $z_2 = (x_0,y_0)$, expressed as follow：

$$A(x_0,y_0) = \int_{z_1 = (x_0,y_0)}^{z_2 = (x_0,y_0)} f(x_0,y_0,z) dz$$

Next, to accumulate the entire Ω, it is equivalent to accumulating each such point (x_0,y_0), which means for any point $(x,y) \in D_{xy}$, there is a cumulative vertical dashed line segment.

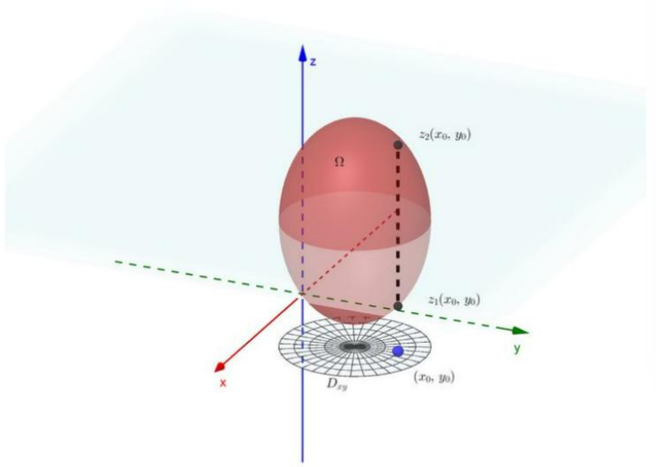
$$A(x,y) = \int_{z_1 = (x,y)}^{z_2 = (x,y)} f(x,y,z) dz$$

$$\iiint_{\Omega} f(x,y,z) dV = \iint_{D_{xy}} A(x,y) dxdy$$

Substituting into the previous formula we can get a formula:

$$\iiint_{\Omega} f(x,y,z) dV = \iint_{D_{xy}} \int_{z_1 = (x,y)}^{z_2 = (x,y)} f(x,y,z) dzdxdy$$

Projecting from several other directions is also a similar calculation method.



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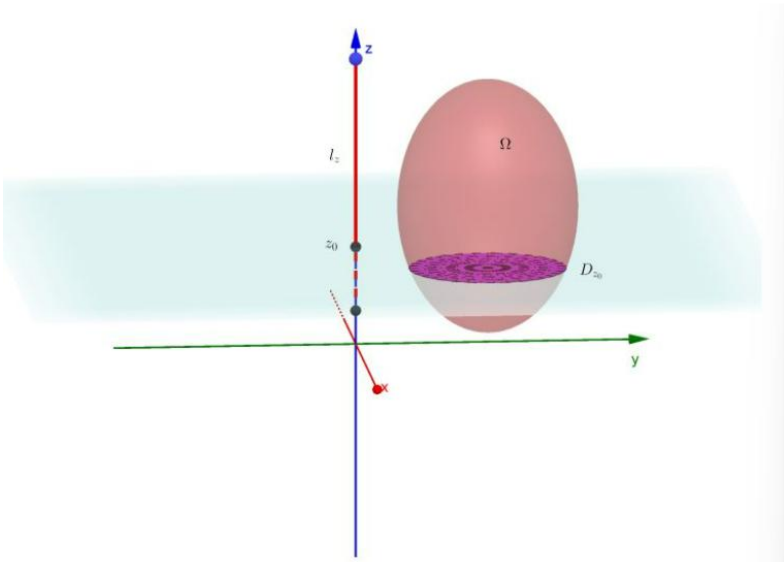
（2）Section Method

The main method is to cut a plane, for example, use $z = z$ section Ω. From the perspective of triple integral, it is to first accumulate the cross-section along the xOy plane direction, and then accumulate all the cross-sectional areas in the plane. Mark the cross-section of Ω at $z = z$ as D_z, and the cross-section of Ω at $z = z$ as l_z. So, as shown in the figure, you need to accumulate purple cross-sections first, and then accumulate each such cross-section on l_z. The cumulative cross-section can be expressed as $A(z) = \iint_{D_z} f(x,y,z)dxdy$. The mass of the whole object can be obtained by accumulation which is $\iiint_{\Omega} f(x,y,z)dV = \int_a^b dz \iint_{D_z} f(x,y,z)dxdy$



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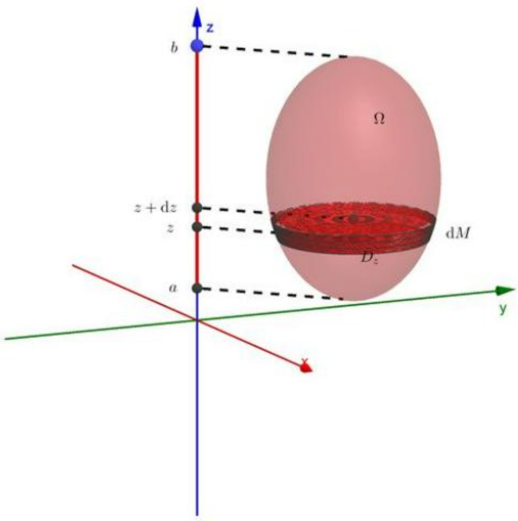
From the perspective of definite integral calculus, it is still the section D_z.

①integral variable $z \in [a,b]$;

②pick point from$[z,z + dz] \subset [a,b]$, the microelement mass is $dM = \left(\iint_{D_z} f(x,y,z)dxdy\right) dz$

③ Then we get the total accumulated mass:

$$\iiint_{\Omega} f(x,y,z)dxdydz = M = \int_a^b dM = \int_a^b dz \iint_{D_z} f(x,y,z)dxdy$$



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In the same way, you can also use the cross-section D_x of $x = x$andΩ, or the cross-section D_y of $y = y$ and Ω, and then convert the triple integral into cumulative integral to calculate.

4. Summary

The concepts and methods of calculus plays an important role in mathematics. The main content is the properties, calculations and applications of derivatives and differentials, etc. Derivatives are local features such as the rate of change and extreme value of the study function, while Integral is the study of the accumulation of small changes from the whole. This paper briefly describes the origin of calculus and related geometric applications, hoping to be helpful to future work.

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