



Original Research Article

Study on the Effect of Material Absorption of Photonic Crystals on Transverse Magnetic Wave Band

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ABSTRACT

Photonic crystals are a major discovery in physics and have an important influence on our present life. The biggest feature of the photonic crystals is it has bandgap which can block photons of a certain frequency, thus affecting the photon movement. This effect is like the influence of the semiconductor body on electrons. Therefore, research and discovery of the photonic crystal is a broad prospect, people has large expectation on the photonic crystal. The emergence of photonic crystals makes it possible for the miniaturization and integration of some aspects of information technology. Study of structure able to determines characteristics, thus the discovery of the photonic crystal structure and function will lay the foundation for the study of its application, in this paper, the study is focus on the research of material absorption of photonic crystal on Transverse Magnetic (TM) wave band. Firstly, the basic knowledge and principle of photonic crystal are introduced. Then, the research was carried out to study the effect of characteristic matrix method on photon crystal TM energy wave through designed experiments and analyzed by computer software.

KEYWORDS: photonic crystal TM band characteristic matrix

1. The basic knowledge and application of photonic crystal

1.1. Introduction

In 1987, S.John and E. Yablonovitch proposed the concept of photonic crystal, and pointed out that the photonic crystal has space-periodic dielectric structure, it is similar with the electric energy gap caused by the semiconductor periodic structure, it will prohibit some of the optical band to spread, and thus forms the photonic band gap. Determination of feature length of the spatial structure of the photonic bandgap is similar to that of the light wave, magnitude is between several hundred nanometers to micrometers. Because this kind of artificial optical material has the special optical properties, it may be applied in the new type of optical devices and thus causes the people's great interest on it.

It is difficult to apply a new scientific outcome into real life, but the future of photonic crystal circuits and devices seems to be sure. If in the short term, the basic application material of photonic crystals in the market appears, there will be a great business prospects. In these applications, highly efficient photonic crystal laser and high brightness light-emitting diodes will account for a large proportion. With the continuous development of society, the improvement of quality of life, information technology is more and more important, the network has become a necessity in people's lives, fiber plays an important role in this, and now 'set-top box' is similar to the decoding signal device in which it used photonic crystal circuits and system. According to the speculation of scientists, within five to ten years, we should produce the first photonic crystal 'diode' and 'transistor'; in science and technology gradually become advance in ten to fifteen years, we should be able to create the first photon crystal logical circuit which then can be used in our lives; in the next twenty-five years, photonic crystal technology becomes more advance, photonic computer which driven by the photonic crystal. Furthermore, photonic crystals can even find role in the jewelry and art market; photonic crystal film can also be used as a security sign. From this point of view, the future of photonic crystals is full of brightness and immeasurable.

2. The classification of photonic crystals

According to the order of the dielectric layer of the photonic crystal, it can be divided into one-dimensional, two-dimensional and three-dimensional photonic crystals. The spatial structure of the one-dimensional photonic crystal was showed as Figure 1.below:

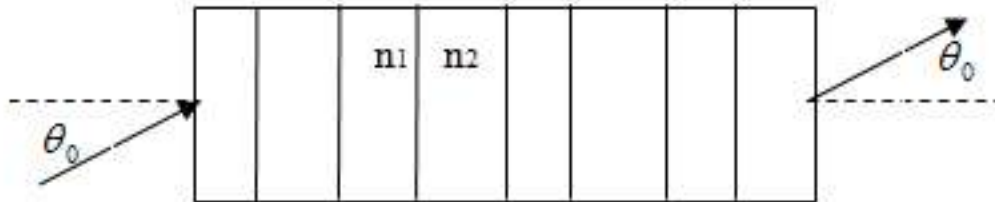


Figure 1. Spatial structure of one-dimensional photonic crystals

One-dimensional photonic crystal is a photonic crystal material with the dielectric refractive index in which it only periodically distributed in one direction in the space. The simplest one-dimensional photonic crystals are typically composed of two layers of dielectric thin films with different refractive indices. The one-dimensional photonic crystals cause the dielectric constant of which perpendicular to the dielectric layer to change periodically with respect to the spatial position, and the dielectric constant of which parallel to the direction of the plane does not change with the spatial position. One-dimensional photonic crystal structure is relatively simple, easy to prepare, easy to study, and it is a strong representation, and the application is very extensive, such as the antisense or permeable membrane in Fabry-Perot cavity optical multilayer.

Two-dimensional photonic crystals refer to materials with photonic bandgap properties in all directions of the two-dimensional space, which are arranged in parallel and evenly by a plurality of dielectric rods. The basic structure of a common two-dimensional photonic crystal is shown in Figure 2, usually a regular crystal system formed by a dielectric column.



Figure 2. Spatial structure of two-dimensional photonic crystals

The dielectric constant of two-dimensional photonic crystal is changing periodically with the spatial position in which it is perpendicular to the medium direction (in both directions), and when parallel to the direction of dielectric rod, it does not change with the spatial position. In order to achieve the regulation of the bandgap range of the two-dimensional photonic crystal frequency, the crystal structure composed of the array of dielectric rods can be changed to make the cross-section in a variety of shapes, such as rectangular, triangular, graphite hexagon and so on.

The common structure of the three-dimensional photonic crystal is the spatial periodic structure of the two media cubes, as shown in Figure 3:

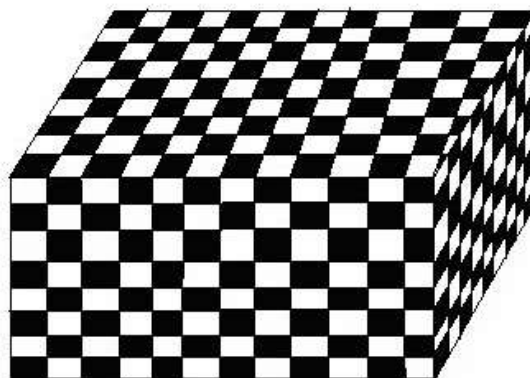


Figure 3. Spatial structure of three-dimensional photonic crystals

Three-dimensional photonic crystal materials have photonic frequency bandgap properties in three-dimensional space, and three-dimensional photonic crystals have a wide application prospect. Due to the limitation in science and technology, the preparation of three-dimensional photonic crystals is more difficult. The first three-dimensional photonic crystal with completed photonic bandgap is created by Yablonovitch of Bell Communications Research, a space periodic structure composed of many face-centered cubes, also known as diamond structures.

3. Photonic crystal characteristics and application

After the emerged of photonic crystals, it become very popular in optical research within a very short period of time. The most important and fundamental feature of photonic crystals is the band gap and conduction band. The study of the bandgap of photonic crystals can control the propagation state of light and suppress spontaneous radiation. For the doped photonic crystal, the defect mode can produce special characteristic such as photonic localization and others.

For the study of the band gap of photonic crystals, we must first understand the basic value of light propagation, that is, transmittance T.

The transmittance T for a single layer of media can be defined as follows:

In the above equation, T is the light transmittance, I_t is the emitting light intensity, I_0 is incident light intensity. That is,

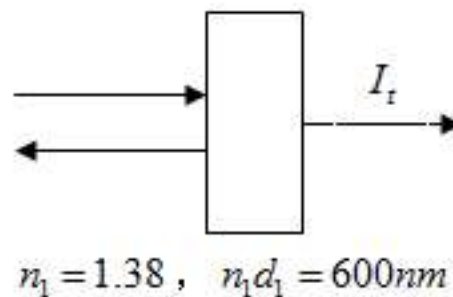


Figure 4.

The transmittance is the ratio of the intensity of the emitted light to the incident light after passing through the medium. (As shown in Figure 4)

Based on the conservation law, we know that the reflectivity R is

In the above formula, R is the reflectance of the medium and T is the transmittance. Figure below (Figure.5) showed the transmission of a single layer of medium for different wavelengths of light.

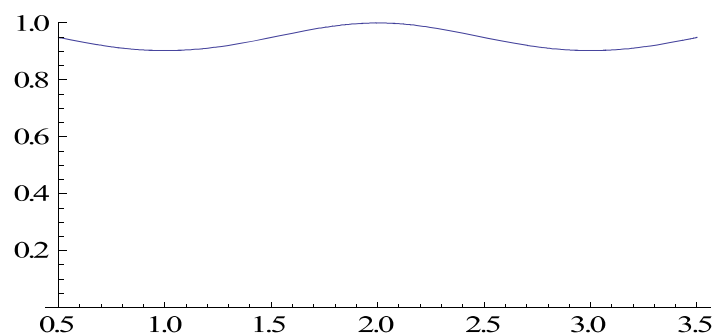


Figure 5. Light transmittance of single layer dielectric

We define the range of transmittance $T = 0$ as the band gap in the photonic crystal, and the transmittance > 0 is the conduction band of the photonic crystal. Usually photonic crystals is more likely to be applied on the absolute regulation of light, thus this paper also focuses on the band gap of photonic crystals, that is, the part that photons cannot be propagated.

The study result showed the transmittance of one-dimensional photonic crystals as shown in Figure 6 below:

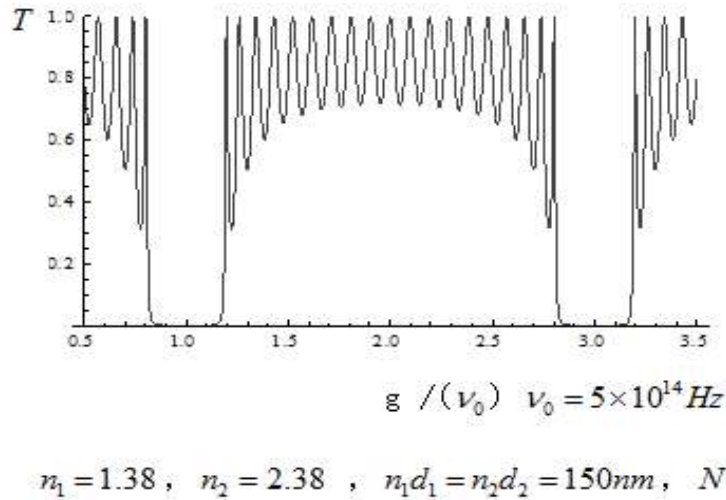


Figure 6. Dimensional photon crystal transmittance

Through the application of photonic crystals, we can artificially control the propagation of electromagnetic waves (elastic waves), while band gap of photonic crystals can be used to develop the anti-electromagnetic radiation film.

In one-dimensional photonic crystals, the characteristics of one-dimensional doping photonic crystals are also special. The basic structure shown in Figure 1.7:

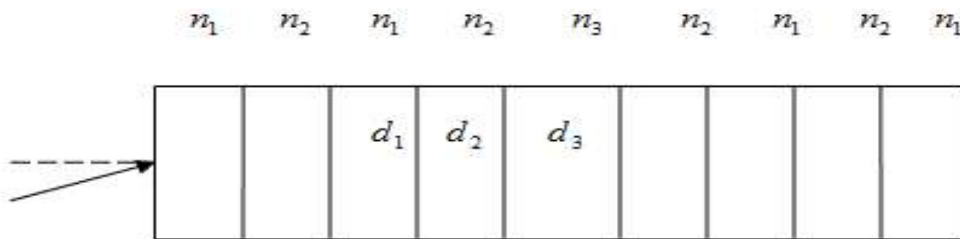


Figure 7. One - Dimensional Photonic Crystals

The addition of a third medium to a regular one-dimensional photonic crystal can form a one-dimensional doped photonic crystal, while keeping most of the properties of a one-dimensional photonic crystal, it is also shows a new characteristic, which is the defect mode.

Figure 8 and Figure 9 show the TE wave and TM wave defect modes with the changes of incident angle and incident wavelength, respectively.

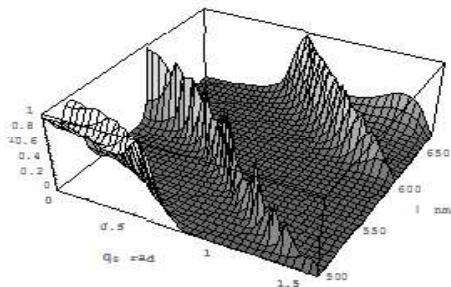


Figure 8. Stereogram of the TE wave defect pattern with incident angle and incident wavelength variation

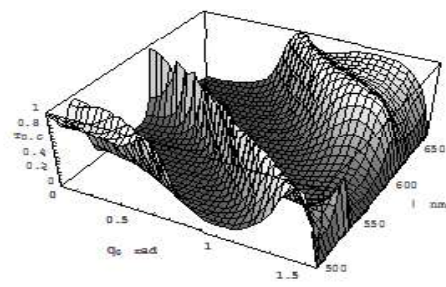


Figure 9. Three-dimensional diagram of TM wave defect mode with incident angle and incident wavelength variation

Figure above showed that there is a very sharp protrusion is formed in the band gap in the both TE wave and the TM wave, forming a very narrow conduction band. Figure 10 showed obviously that the defect mode of doped photonic crystals.

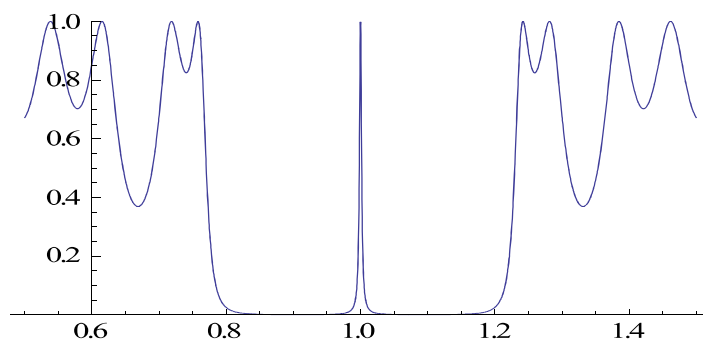


Figure 10. The Variation of Elastic Wave Defect Mode with Frequency

Previous research literature showed that the frequency of the one-dimensional doped photonic crystal defect mode is determined by the thickness of the doped medium. The frequency width of the defect mode is determined by the thickness and the refractive index of the dielectric film.

4. The total reflection tunneling properties of photonic crystals

The total reflection tunneling of one - dimensional photonic crystals is a new phenomenon found in the last two years. In order to obtain the total reflection tunneling of one-dimensional photonic crystals, the response curve of the transmittance of the light wave from the zinc sulfide to the single interface of magnesium fluoride with the incident angle is observed first. Since the refractive index (n) of zinc sulfide is greater than the refractive index of magnesium fluoride (n'), thus the light on the single interface will appear full reflection phenomenon, the total reflection angle is θ_c . Figure 7 showed that the transmittance decreases with the increase of the incident angle when the incident angle is less than the total reflection angle, and the transmittance decreases to 0 rapidly when the incident angle approaches the full reflection angle. When the angle of incidence is greater than the full reflection angle, the transmittance is always zero, that is, the full reflection phenomenon occurs, and the light cannot enter the magnesium fluoride from the zinc sulfide in the case of greater than the total reflection angle.

Further study is to determine the case of light incident in one-dimensional photonic crystals, one-dimensional photonic crystals are alternately arranged by magnesium fluoride and zinc sulfide. The medium of the incident space and the exit space is also zinc sulfide. In this case, when the light is incident on the one-dimensional photonic crystal, the total reflection angle is θ_c . The response curve of the transmittance of the one-dimensional photonic crystal with incident angle is calculated as shown in Fig 11. Fig. 11 showed that when the incident of light on the one-dimensional photonic crystal, there are five obvious transmission peaks in the range of the angle of incidence which is greater than the full reflection angle, which indicates that the light wave can penetrate photonic crystals when the light wave is larger than the full reflection angle in photonic crystals. This phenomenon is called 'the total reflection tunneling effect of photonic crystals'.

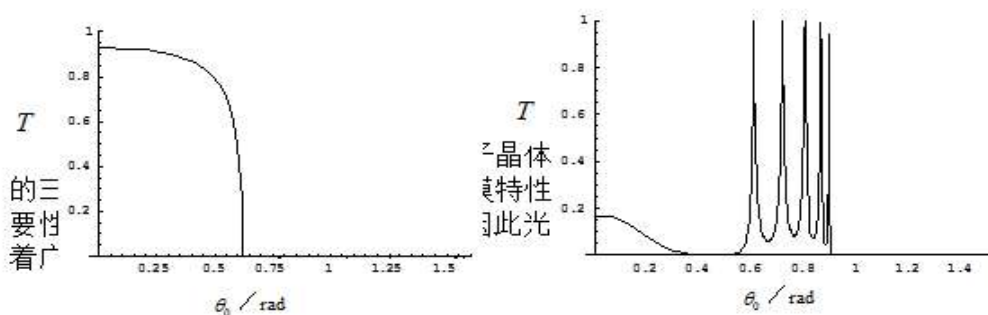


Figure 11. 7 The response curve of the transmittance at the single interface with the incident angle

Figure 12. Curve of the total reflection of the photon crystal with the incident angle

It is also found that the number of total reflection tunneling peaks of one-dimensional photonic crystals is related to the number of cycles of one-dimensional photonic crystals, and the frequency of total reflection tunneling peaks decreases with the increase of the periodic optical thickness of photonic crystals. The discovery of the total reflection tunneling effect of photonic crystals has opened up a new research topic for photonic crystal research. And the total reflection tunneling peak of the photonic crystal has excellent comb filter characteristic, which provides a theoretical basis for designing a new high quality photonic filter by using the total reflection tunneling effect of photonic crystals.

The concept of photonic crystal and the commonly used research method of photonic crystal are introduced. Three important characteristics of one - dimensional photonic crystal are discussed, namely band characteristic, defect mode and total reflection tunneling. The use of these important properties of photonic crystals can effectively control the transmission of photons. Therefore, photonic crystal has a wide application prospect in optical signal transmission and optical signal control.

Chapter 2: A research method of photonic crystal theory - feature matrix method

The most important characteristic of photonic crystals is the photonic band gap, the optical materials and devices such as optical fiber, optical waveguide, full reflection mirror, filter, polarizer and so on are applying this characteristic where the photons cannot propagate through the band gap. When the defective medium is added to the photonic crystal, the propagation of the photon in the photon crystal can be localized to suppress or enhance its spontaneous radiation, so that the photonic crystal can be used to produce high efficiency and zero threshold laser, high quality laser resonator and efficient light-emitting diodes. However, to achieve the application of photonic crystals in these fields, we must know the band structure of photonic crystals, to know its band structure, we must select the appropriate research methods. At present, there are three main methods for the study of photonic crystal band structure: feature matrix method, plane wave expansion method, multiple scattering method and so on.

1. The method of propagation of the photon during medium absorption

For the media with absorption, in order to simultaneously describe its refraction and absorption of light waves, it has to be introduced the concept of complex refractive index and complex wave number:

$$\hat{n} = n(1 + i\kappa) \quad , \quad \hat{k} = \frac{\omega}{c} \hat{n}$$

Whereis the refractive index of the medium to describe its refraction to the light, is the extinction coefficient of the medium to describe its absorption to the light, c being the speed of light in the vacuum, and is the circular frequency of the light. The light waves propagating in the absorption medium satisfy:

$$\nabla^2 \vec{E} + \hat{k}^2 \vec{E} = 0$$

$$\vec{E} = \vec{E}_0 e^{i(\hat{k} \cdot \vec{r} - \omega t)}$$

is the unit vector for the direction of light propagation. Combined with formula (1):

$$\vec{E} = \vec{E}_0 e^{\frac{-\omega}{nc} \vec{r} \cdot \vec{s}} e^{i\omega(\frac{n}{c} \vec{r} \cdot \vec{s} - t)}$$

It can be seen from (4) that the light wave propagating in the absorption medium is the attenuation wave. However, it can be seen from (2) and (3) that the equation and the corresponding solution in the absorption medium satisfy the equation and the corresponding solution in the transparent medium is exactly the same, but only the number of complex waves replaces the wave number. Therefore, in dealing with the propagation of light in the absorption medium, it is only necessary to change the refractive index in the formula corresponding to the transparent medium to the complex refractive index, and the wave number can be solved by changing the number of waves.

2) the derivation of the characteristic matrix method

According to the thin film optics theory, we can use a 2×2 matrix to represent the transmission properties of light in each layer of the medium, which becomes the characteristic matrix of light propagation. For the first layer of media, the characteristic matrix is:

$$M_j = \begin{bmatrix} \cos \delta_j & -\frac{i}{p_j} \sin \delta_j \\ -ip_j \sin \delta_j & \cos \delta_j \end{bmatrix}$$

Where $\delta_j = (2\pi/\lambda) \cos \theta_j d_j$ is the optical thickness of the dielectric layer and θ_j is the angle of light between medium layer and the normal direction of the interface, and λ is the wavelength of the incident light. Among them

$$\begin{cases} p_j = n_j \cos \theta_j & \text{TE 波} \\ p_j = \cos \theta_j / n_j & \text{TM 波} \end{cases}$$

(1) The overall characteristic matrix of one-dimensional photonic crystals is:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = (M_1 M_2)^N$$

When the light from the air emit into the photonic crystal, the reflection coefficient is:

$$R = |r|^2$$

The reflectance:

$$T = 1 - R$$

The transmittance

(2) The overall characteristic matrix of one-dimensional doping photonic crystals is:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = (M_1 M_2)^N M_3 (M_2 M_1)^N$$

When the light from the air emit into the photonic crystal, the reflection coefficient is:

$$r = \frac{(M_{11} + M_{12} p_h) p_0 - (M_{21} + M_{22} p_h)}{(M_{11} + M_{12} p_h) p_0 + (M_{21} + M_{22} p_h)}$$

The reflectance:

$$T = 1 - R$$

The transmittance of one-dimensional photonic crystals can be calculated by using the above formula, and thus the studies on bandgap and filter characteristics can be carried out.

Chapter 3: The research on the influence of absorbing material on one - dimensional photonic crystal TM wave energy band

1. one-dimensional photonic crystal structure and light propagation characteristics

One-dimensional photonic crystals are photonic crystal materials with periodic distribution of dielectric refractive index in one direction of space. The basic structure of a one-dimensional photonic crystal is composed of two kinds of dielectric films with different refractive indices. This structure makes the dielectric constant of photonic crystal which perpendicular to the dielectric layer periodically changes with the spatial position, whereas the dielectric constant which parallel with the dielectric layer is constant value. In this paper, we focus on the study of simple structure of one-dimensional photonic crystal, the structure is alternating composed of two types of material with the refractive index of and the thickness are and , respectively, and the thickness of a cycle is, this structure is similar with the multilayer dielectric film, The spatial refractive index is when the incident light wave is perpendicular to the photonic crystal interface, it is as shown in Figure 13:

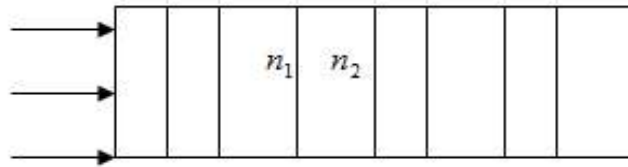


Figure 13. The structure of one - dimensional photonic crystals

When the light is propagated inside it, the bandgap and conduction band will appear, and this is the propagation characteristics of the simple structure of the one-dimensional photonic crystal and it also the main point of this paper.

2. Influence curve of the TM wave absorption band with the changes of frequency (2D)

When $n_1 = 1.38$ (magnesium fluoride), $n_2 = 2.38$ (zinc sulfide) and the incident angle is $\theta = 0.5\text{rad}$, the condition of the elimination coefficient, k at 0, 0.01, 0.03 was observed when reflectivity, R changes with frequency, g .

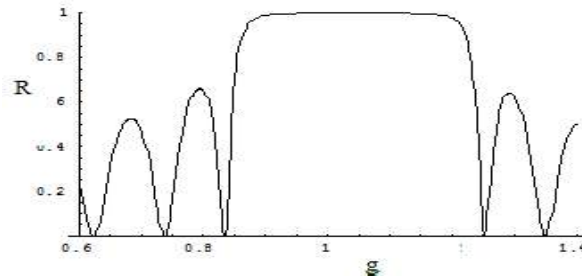


Figure 14. $k=0$

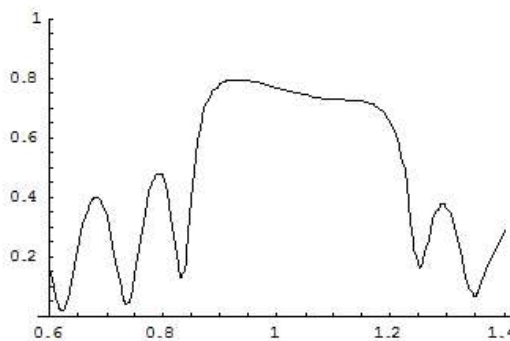
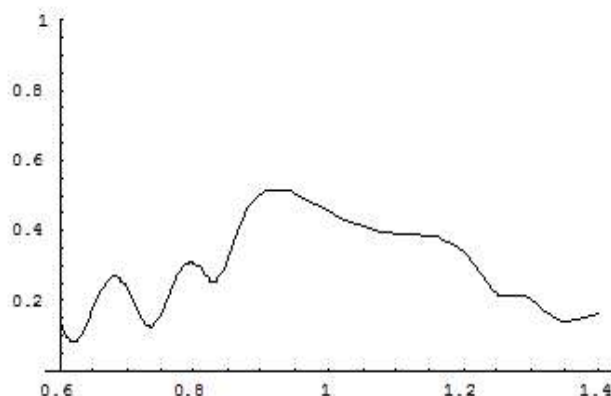


Figure 15. $k=0.01$ Figure 16. $k=0.03$

The figure above showed that when $K = 0.00$ (Figure 3.2), that is, there is no absorption at this time, the reflectivity R is 1 when frequency is in interval of 0.8-1.2, then a band gap is appear, and the band distribution is very average.

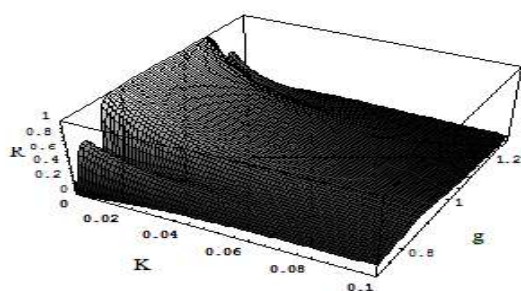
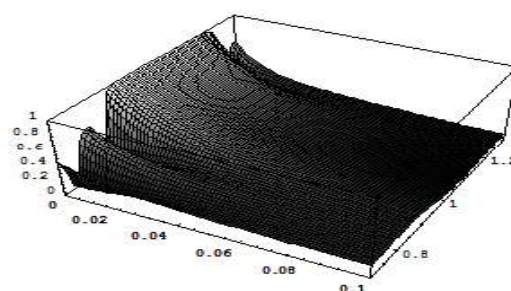
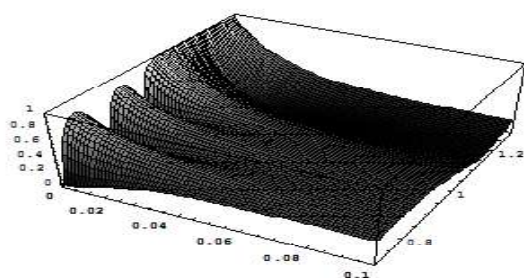
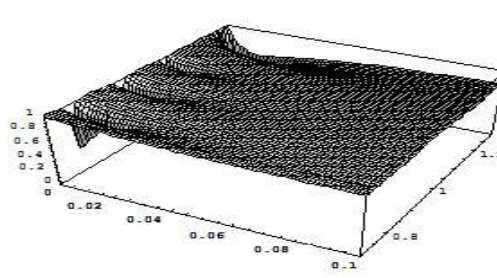
When $k = 0.01$ (Figure 3.3), the band gap reflectivity decreases, the peak is about 0.8, the band is still obvious.

When $k = 0.03$ (Figure 3.4), the peak of the reflectivity dropped to about 0.6, the band gap is no longer obvious and loss its function.

According to the above data, it showed that with increased of the absorption coefficient k value, the band gap will become weaker, the band gap reflection gradually reduced, the peak becomes smaller, the width also becomes narrow, and the peak almost disappeared at last.

3. Influence curve of TM wave absorption band with changes of the absorption coefficient and frequency (3D)

When $\theta = 1.38$ (magnesium fluoride), $\theta = 2.38$ (zinc sulfide), the changes condition when the incident angle is θ , the reflectivity R when the absorption coefficient, k is in the range of (0-0.1), the frequency, g is in the range of (0.6-1.4) was observed. The experiment was carried on at $\theta = 0, 0.5 \text{ rad}, 1 \text{ rad}, 1.5 \text{ rad}$.

图 3.5 $\theta=0$ 图 3.6 $\theta=0.5\text{rad}$ 图 3.7 $\theta=1\text{rad}$ 图 3.8 $\theta=1.5\text{rad}$

The above figures were analyzed and showed that when $k = 0$ (no absorption), the reflectance of TM wave is also 1 when incident angle, θ is 0, 0.5 rad, 1 rad, 1.5 rad, the band gap appear in all angle. When k increases to 0.05, the reflectivity decreases to about 0.41, the top of the band gap is not smooth, the edge is also increasingly blurred.

When $\theta = 0$, the band gap is clear, the edge changes are very steep and obvious, and the peaks on both sides have the same trend, and when $\theta = 0.5$ rad and 1 rad, although a clear band gap was still observed, but in interval of 1-1.4 of normalized frequency, the band edge gradient slowed down, it almost no changes can be seen, when $\theta = 1.5$ rad, the reflectivity changes more and more gentle, the peak almost unchanged.

When k is gradually increased, the corresponding transmission peak value of TM wave are also gradually increased when incident angle θ is 0, 0.5 rad, 1 rad, 1.5 rad.

As k increases gradually, the width of the band gap frequency decreases.

Regardless of how the k and the angle of incidence change, the peak position is at the position of frequency $g = 1$.

When k is constant, the transmission peak increases with the increased of the incident angle, and the band gap reflectivity increases with the increased of the incident angle, and the frequency width decreases with the increased of the incident angle.

4. Influence curve of TM wave absorption band with the cycle thickness and frequency changes (3D)

When $n = 1.38$ (magnesium fluoride), $n = 2.38$ (zinc sulfide), the changes was observed whe the incident angle is $\theta = 0.5$ rad, the absorption coefficient k is changed, the reflectivity R varies with the cycle thickness X (0.8-1.2), and the frequency g is (in the range of = 0.5-1.5) rang. The experiment was carried on at $k = 0, 0.01, 0.02, 0.03$

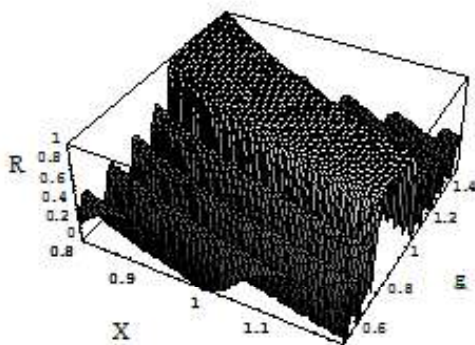


图 3.9 $k=0$

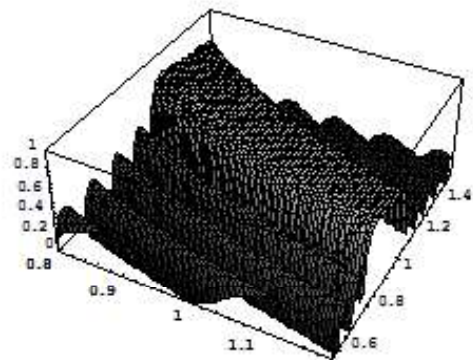


图 3.10 $k=0.01$

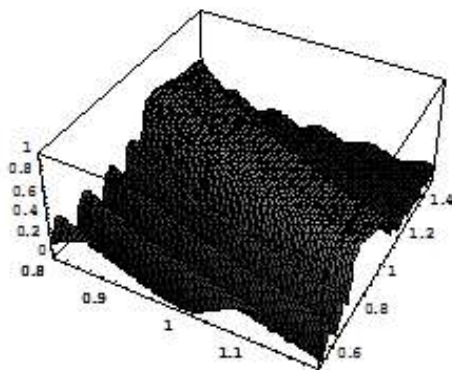


图 3.11 $k=0.02$

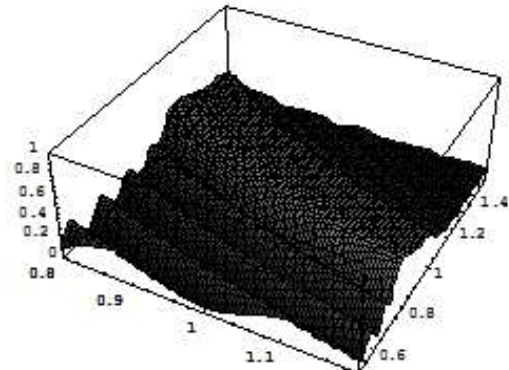


图 3.12 $k=0.03$

The figure showed that when $k = 0$, the reflectivity peak corresponding reflectance is 1, and a very obvious photonic bandgap occurs at the frequency of 0.8-1.2, and the width of the bandgap frequency is the largest.

When $k = 0.01$, the reflectivity peak corresponding reflectance is reduced to about 0.8 in the frequency of 0.8-1.2, and the photonic bandgap is still more obvious.

When $k = 0.02$, the reflectivity peak corresponding reflectance is reduced to about 0.6 in the frequency of 0.8-1.2, although it can see the photonic band gap, but it has begun to become blurred.

When $k = 0.03$, the reflectivity peak corresponding reflectance is reduced to about 0.5 in the frequency of 0.8-1.2, and the band gap of the photonic crystal has disappeared.

From the above analysis, it can be concluded that when the k is gradually increasing, the reflectivity is decreased, the peak is getting lower and lower, and the reflectivity corresponding to the band gap is getting lower and lower. When k is constant, with the increase of the cycle thickness, the reflectivity decreases gradually, the corresponding reflectance peak is getting lower and lower, the band gap frequency range becomes more and more narrow.

5. research conclusions

By changing the incident angle and the absorption coefficient, the three-dimensional 3D image of the band of the one-dimensional photonic crystal TM wave with the absorption coefficient and the cycle thickness and frequency is plotted by Wolfram Mathematica software. Through the comparison analysis of the 3D image, the influence of the absorption coefficients on the energy band of the one-dimensional doped photonic crystal is as follows:

1. The extinction coefficient of the impurity has a significant effect on the transmission peak of the TM wave. Absorption coefficient $K = 0$, that is, no absorption state, the transmission is most obvious, the reflectivity, $R = 1$, there is a band gap, its shape similar to the rectangular, band gap waveform is more obvious and symmetrical. With the increase of the absorption coefficient k , the band gap will become weaker and weaker, the band gap reflection will gradually decrease, the peak becomes smaller and the width will narrow, and finally the peak will disappear almost.

2. When the extinction coefficient increases to 0.03, the corresponding peak reflectivity decreases, the band gap becomes narrows, the top is not, the future bandgap gradually disappear.

3. When the absorption coefficient k increases, the reflected wave width increases and the half width is gradually reduced.

4. When the extinction coefficient is constant, the transmission peak moves towards to the direction of high frequency with the increase of the incident angle, and when the incident angle changes, the peak of its reflectance is always 1.

5. When the extinction coefficient is constant, the transmittance decreases with the increase of the cycle thickness, and the reflection peak also decreases with the increase of the cycle thickness, and the band gap is gradually disappears.

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