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Extrapolation-Newton Iterative Solution for Nonlinear Equation

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Abstract: In this paper, the solution of nonlinear equation is evaluated using Newton method, and then an extrapolation procedure with Aitken method is applied to improve and accelerated the Newton solution by reducing the number of iterations in order to find the roots of the nonlinear equation with minimum iterations. Several examples are solved using Newton Aitken technique. The comparison based on depending on the number of iterations.

Keywords: Extrapolation method, Newton method, Aitken algorithm
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1. Introduction

Some iterative ways are obtained using numerical methods to solve many nonlinear problems occurring in artificial satellites[1-7], the solar system, ceramics material, and many other applications[8-13]. One of the efficient and simple methods that can be used to solve nonlinear equations is Newton method, which has second order convergence. Many researchers for treating nonlinear equations suggest modified Newton method[14-19]. Some other methods for numerical solution of nonlinear equations can be found in[20-36]. In this article an extrapolation technique with the help of Aitken method are proposed to improve the Newton method solution.

2. Aitken Extrapolation Procedure

In order to accelerate the rate of convergence of a sequence, Aitken extrapolation procedure will be proposed.

Assume that a sequence $x_o, x_1, \dots, x_k$ is given then a new sequence $E_o, E_1, \dots, E_k$ can be presented as follows

$$E_k = x_k - \frac{(x_{k+1}-x_k)^2}{(x_{k+2}-2x_{k+1}+x_k)} \quad 1$$

or equivalently

$$E_k = x_k - \frac{\Delta x_{k+1}}{\Delta^2 x_k} \quad 2$$

where Δx_{k+1} is forward difference operator

$$\Delta x_{k+1} = x_{k+2} - x_{k+1}$$

or $\Delta^2 x_k = \Delta x_{k+1} - \Delta x_k$

Now the extrapolation sequence is suggested to be

$$E_k = x_{k+2} - \frac{\Delta x_{k+1}}{\Delta x_{k+1} - \Delta x_k}, k = 0, 1, 2, \dots \quad 3$$

Note that, in this work the sequence $x_1, x_2, \dots$ are obtained using Newton method and x_o is the initial value.

3. The algorithm ANE

Aitkin-Newton extrapolation (ANE) algorithm is adopted and summarized by the following steps

Step 1: Identify the equation $f(x) = 0$.

Step 2: select the initial value x_o

Step 3: Use Newton method to obtain x_1 and x_2

$$x_1 = x_o - \frac{f(x_\circ)}{f'(x_\circ)} \quad , \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Step 4: Use extrapolation technique to obtain $E_\circ$

$$E_o = x_2 - \frac{(x_2 - x_1)^2}{(x_2 - 2x_1 + x_o)}$$

Step 5: set = 3 , and obtain x_3, E_1

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$E_1 = x_3 - \frac{(x_3 - x_2)^2}{(x_3 - 2x_2 + x_1)}$$

Step 6: set $k = 4$: n

$$x_k = x_{k-1} - \frac{f(x_{k-1})}{f'(x_{k-1})}$$

$$E_{k-2} = x_k - \frac{(x_k - x_{k-1})^2}{(x_k - 2x_{k-1} + x_{k-2})}$$

Step 7: If $|E_k - E_{k-1}| < \epsilon$ stop else go to step 5 where ϵ is input Tolerance value.

4. Numerical Examples

The effectiveness of the proposed extrapolation algorithm is illustrated by solving five examples.

Example 1: $f(x) = (e^x - sin(x))^2$

The above test function is solving using Newton method and the ANA. Table 1 shows that the efficiency of the extrapolation procedure obtains better results of convergence as compared to the Newton method. It observes that the computations converge in less than three iterations where tolerance value $\epsilon = 0.00001$.

Example 2: $f(x) = (x^5 - x^3 - 2)^2$

The above test function is solving using Newton method and the ANA. Table 2 shows that the efficiency of the extrapolation procedure obtains better results of convergence as compared to the Newton method. It observes that the computations converge in less than eight iterations where tolerance value = 0.00001 .

Example 3: $f(x) = e^{2x} - x - 6$

The above test function is solving using Newton method and the ANA. Table 3 shows that the efficiency of the extrapolation procedure obtains better results of convergence as compared to the Newton method. It observes that the computations converge is less than one iteration where tolerance value $\epsilon = 0.000000001$

k	The solution using Newton method	The solution using extrapolation
1	-3.1374	-3.1647
2	-3.1602	-3.1830
3	-3.1716	-3.1830
4	-3.1773	
5	-3.1802	
6	-3.1816	
7	-3.1823	
8	-3.1827	
9	-3.1828	
10	-3.1829	
11	-3.1830	
12	-3.1830	

Table 1. Results of Newton method and extrapolation algorithm with $x_o = -3$

k	The solution using Newton method	The solution using extrapolation
1	1.2925	1.3118
2	1.3131	1.3443
3	1.3255	1.3472
4	1.3333	1.3472
5	1.3383	1.34755
6	1.3415	1.34762
7	1.3437	1.34762
8	1.3451	1.3478
9	1.3460	
10	1.3466	
11	1.3470	
12	1.3473	
13	1.3475	
14	1.3476	
15	1.3477	
16	1.3478	
17	1.3478	

Table 2. Results of Newton method and extrapolation algorithm with $x_o = 1.2925$

k	The solution using Newton method	The solution using extrapolation
1	0.97	0.97087002
2	0.970870836	
3	0.97087002	

Table 3. Results of Newton method and extrapolation algorithm with $x_o = 0.97$

k	The solution using Newton method	The solution using extrapolation
1	1	1.468202904
2	1.54207919	1.45457555
3	1.4565461937	1.454618
4	1.4546199343	
5	1.4546189292	
6	1.454618929	

Table 4. Results of Newton method and extrapolation algorithm with $x_o = 1$

k	The solution using Newton method	The solution using extrapolation
1	1.5	1.9476366431
2	3.0765582	1.8941563937
3	1.9105066	1.8954931645
4	1.8954622	1.8954931645
5	1.89549427	
6	1.89549426	
7	1.89549426	

Table 5. Results of Newton method and extrapolation algorithm with $x_o = 1.5$

Example 4: $f(x) = x^2 - cosx - 2$

The above test function is solving using Newton method and the ANA. Table 4 shows that the efficiency of the extrapolation procedure obtains better results of convergence as compared to the Newton method. It observes that the computations converge in less than three iterations where tolerance value $\epsilon = 0.000000001$

Example 5: $f(x) = x - 2\ sinx$

The above test function is solving using Newton method and the ANA. Table 5 shows that the efficiency of the extrapolation procedure obtains better results of convergence as compared to the Newton method. It observes that the computations converge in less than four iterations where tolerance value $\epsilon = 0.0000001$

5. Conclusion

A new modification of Newton method is presented in this work, which include Aitkin extrapolation method. The method requires one evaluation of both the function and its first derivative per iteration. By introducing Aitkin extrapolation procedure, a fast convergence is achieved. This fact is illustrated by considering some numerical results. The obtained results show that the numbers of iterations are reduced very rapidly.

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