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Numerical Solution of Nonlinear Equations Using Multi-step Homotopy Perturbation Algorithm

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Abstract: In this article, some new improvement for fifth order multistep method [1] is adopted with the idea of homotopy perturbation procedure to reach the solution of nonlinear equations in minimum number of iterations. Suggest way to identify a start system of the proposed method is also included within this work. The obtained results are compared in terms of the iterations number and the application of the presented method based on studding several examples.

Keywords: Multistep method, homotopy perturbation procedure, start system, nonlinear equation.

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1. Introduction

It is well known that much class of problems are formulated as nonlinear equations which appear in different discipline of both pure and applied sciences as well as in engineering applications[2-15]. For example finding the true anomaly from mean anomaly value and the eccentricity for a planet in an elliptical orbit around the sun will lead to solve Kelper's equation, which has a unique solution. Several numerical methods have been derived for solving such equations because of its importance in celestial mechanics[17-24]. In general, the nonlinear equations f(t)=0 can be solved using several approximate techniques, which have been proposed by many authors[25-33]. Another important method for treating the nonlinear equation is homotopy method[34-36]. Attention is given to improve some iterative methods for solving nonlinear equations based on homotopy perturbation method. In this paper, an improvement is suggest for solving nonlinear equation based on the multistep method[1] and the homotopy perturbation method. Some example are solved to illustrate the efficiency of this method and a comparison is made depends on the number of iterations.

2. The Method

The basic idea of linear homotopy is given in the following definition.

Definition 1.1

Consider a non-linear algebraic equation $f(x) = 0$, then convex homotopy for the function $H(x,\lambda): R \times [0,1] \to \Re$ is

$$H(x,\lambda) = (1 - \lambda)p(x) + \lambda\, q(x) = 0 \qquad (1)$$

where λ is an embedded parameter and $\lambda \in [0,1]$;

$p(x)$ is the start system;

$q(x) = f(x)$ is the target system;

$H(x,0) = p(x)$ & $H(x,1) = q(x) = f(x)$.

The basic ways to identify a start system $p(x)$ of a linear homotopy are

1. $p(x) = x - x_0$,

$$H(x,\lambda) = (1 - \lambda)p(x) + \lambda\, q(x) = 0; \qquad (2)$$

where is an initial approximation of Eq. 2

2. $p(x) = q(x) - q(x_0)$,$H(x,\lambda)$ $[\, q(x) - q(x_0)\,]+$ $\lambda\, q(x) = 0$ (3)

3. $p(x) = x^n - c,\ H(x,\lambda) = (1 - \lambda)\,(x^n - c) +$ $\lambda\, q(x) = 0$ (4)

3. Results and Discussions

A. Algorithm 1

Step 1: Find the initial value x_0, by setting $p(x) = x^n - c = 0 \rightarrow x_0$.

Step 2: Evaluate y $= \left(x - \frac{f(x)}{\acute{f}(x)}\right)$;

Step 3: Evaluate z=$\left(y - \frac{2f(y)\ f'(y)}{2((\acute{f}(y))^2 - f(y)\ f''(y)}\right)$

Step 3: Evaluate new $x = y - \left(\frac{2[\,f(y) + f(z)]\,f'(y)}{2((f'(y))^2 - [f(y) + f(z)]\ f''(y)}\right)$

B. Algorithm

Step 1: Identify $q(x) = f(x) = 0$

Step 2: Identify $p(x)$, such as $p(x) = x^n - c$

where c is a any real number, and n be the highest power of x or, $p(x)$ be a part of $f(x)$ with trivial solution (s),

Step 3: Identify $p(x)$, such as $p(x) = x - x_0$.

Step 4: Find the initial value x_0, by setting $p(x) = x^n - c = 0 \rightarrow x_0$.

Restart, $q: = x \rightarrow x^2 - (1-x)^5$; $p: = x \rightarrow x^5 - 1$;
$fsolve\ (q(x))$;
$N := fsolve\ (p(x))$;

Step 5: simplify $H(x,\lambda) = (1 - \lambda)\ (x^n - c) + \lambda\, q(x)$

Such as,
$H := x \rightarrow (1 - \lambda)\, p(x) + \lambda\, q(x)$; simplify $(H(x,\lambda))$;

Step 6: iterate $H(x,\lambda) = (1 - \lambda)\ (x^n - c) + \lambda\, q(x)$

where $\lambda \epsilon [0,1]$ e.g . 0.2, 0.4, 0.6, 0.8, 1.0 by using the (three steps in algorithm 1)

DH: = D(H); simplify (DH (x,λ));

New t : = y → evalf $\left(x - \frac{H(x,\lambda)}{H'(x,\lambda)}\right)$;

New : = $Z \rightarrow evalf$
$\left(y - \frac{2H(y,\lambda)\ * H'(y,\ \lambda)}{2((H'(y,\ \lambda))^2 - H(y,\lambda) * H''(y,\lambda)}\right)$;

New: = $x \rightarrow evalf$
$\left(y - \frac{2[\,H(y,\lambda) + H(z,\lambda)] * H'(y,\ \lambda)}{2((H'(y,\ \lambda))^2 - [H(y,\lambda) + H(z,\lambda)] * H''(y,\lambda)}\right)$;

Step 7: simplify $H(x,\lambda) = (1 - \lambda)\ (x - x_0) + \lambda\, q(x)$

Such as,
$H := x \rightarrow (1 - \lambda)\, p(x) + \lambda\, q(x)$simplify $(H(x,\lambda))$;

Step 8: iterate $H(x,\lambda) = (1 - \lambda)\ (x - x_0) + \lambda\, q(x)$

where $\lambda \epsilon [0,1]$ e.g . 0.2, 0.4, 0.6, 0.8, 1.0 by using the (three steps in algorithm 1)

DH: = D(H); simplify (DH (x,λ));

New $= \left(x - \frac{H(x,\lambda)}{H'(x,\lambda)}\right)$;

New $Z = \left(y - \frac{2H(y,\lambda)\ * H'(y,\ \lambda)}{2((H'(y,\ \lambda))^2 - H(y,\lambda) * H''(y,\lambda)}\right)$

New $x = \left(y - \frac{2[\,H(y,\lambda) + H(z,\lambda)] * H'(y,\ \lambda)}{2((H'(y,\ \lambda))^2 - [H(y,\lambda) + H(z,\lambda)] * H''(y,\lambda)}\right)$

C. Numerical Examples and analysis

Three test examples are solved to illustrate the efficiency of the multistep homotopy perturbation method and the results are compared against the method derived in [1]. The test examples will be

(a) $f(x) = x^3$ – 2x-5
(b) $f(x) = 5x^3$ $-x\ e^x$-1
(c) $f(x) = x^2$ +8x -9

The number of iterations to reach the solution with an operated initial value x_0 against results presented in [1] are listed in **Table 1**-**Table 3** for the above three test examples.

Function	x_0	[1]	root	λ	$x^n - c$	root	$x - x_0$	root
$f(x) = x^3 - 2x - 5$	1.709975946676697	6	2.0945 51481542 327	0.2	5	1.7878981 67258076	5	1.988508065502361
				0.4	5	1.8655302 51588451	6	2.047919662442417
				0.6	6	1.9426336 47840137	6	2.072621067850605
				0.8	6	2.0190213 69504220	6	2.086086860148989
				1	6	2.0945514 81542327	6	2.094551481542327

Table 1. Results for function $f(x) = x^3 - 2x - 5$

Function	x_0	[1]	root	λ	$x^n - c$	root	$x - x_0$	root
$f(x) = 5x^3 - xe^x - 1$	1	6	0.837177130769307	0.2	5	0.936338813941552	4	0.897261278095561
				0.4	5	0.897758681664413	6	0.867501472503153
				0.6	6	0.871396981307035	6	0.852510573002858
				0.8	6	0.852056582284315	6	0.843366251980199
				1	6	0.837177130769307	6	0.837177130769307

Table 2. Results for function $f(x) = 5x^3 - xe^x - 1$

Function	x_0	[1]	root	λ	$x^n - c$	root	$x - x_0$	root
$f(x) = x^2 + 8x - 9$	3	5	1	0.2	5	2.304834939252005	5	1.549834435270750
				0.4	5	1.800000000000000	6	1.255206074732157
				0.6	6	1.441874542459709	6	1.123568514581633
				0.8	6	1.186342439892262	6	1.048550521643719
				1	5	1	5	1

Table 3. Results for function $f(x) = x^2 + 8x - 9$

4. Discussion

The numerical results are demonstrated that the proposed multistep homotopy perturbation method based on certain way to produce an start value for initial value converges better than the fifth order three-step method in[1].

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