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Certain coefficient andFejerGap series about of homogeneous and non-homogeneous

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Abstract:To: NevanlinnaTheory in complex differential equation field in has widely of Application, which use the theory research complex linear differential equation meromorphic solution of growth and value distribution and coefficient of growth between the relationship is complex differential equation in the field of important topic. due to incomplete series has some special properties when gap series as an equation coefficient when these properties can be play role. so we can be combined with GAP series of definition and properties research complex linear differential equation meromorphic solution of properties. in this paper in we useNevanlinnaTheory and combinedFej 'ErGap series of definition and properties of a class of homogeneous and non-homogeneous high-order complex linear differential equation the research. When equation of a coefficient andFej 'ErGap series about and the rest of the coefficient for the entire function or meromorphic function when get the equation meromorphic solution of growth level of estimation promotion and improved the previous studies have been results.

Keywords:Complex linear differential equation;NevanlinnaTheory;Fej 'ErGap series; iterative level; iterative Style

1. Introduction

In this paper in we "with value distribution theory in standard mark[1-3]. For enough bigR(0;∞)AndP N={1;2; In in ;⋯}Remember

Log_1R= LogR;$Log_{P1}R$= Log ($Log_P R$);Exp_1R= ExpR;$Exp_{P1}R$= Exp ($exp_P R$);

And provisions

Log_0R=R= Exp_0R;$Log_{-1}R$= ExpR;$Exp_{-1}R$= Log_1R:

We also need to use measure and density of definition[4]As follows: Collection E(0;∞)Of line measure Definition Formula Formula

Complex Plane on the meromorphic function we also introduces the following definition. If no special (we agreedPN.

In this paper, we will introduce the background knowledge related to the main results of this paper.

$F^{(K)}A_{K-1.}(Z)F^{(K-1)}\cdots A_{1.}(Z)F' \; A_0(Z)F=F(Z)$ (1)

The relationship between the growth of solutions and the distribution of values and the growth of coefficients is an important aspect.Assume that a coefficient of the equation controls, for example(Iteration)Level or(Iteration)Type strictly greater than other Coefficients(Iteration)Level or(Iteration)Then, the relationship between the growth of the equation solution and the growth of the coefficient is obtained.(Iteration)Level or maximum(Iteration)Type, then

Other conditions must be added to get the corresponding conclusion.$A_0(Z)$The ratio coefficient of results obtained when acting as a control$A_D(Z)$(D J= 0)The results are more accurate when it comes to control.Therefore, many scholars further consider

$A_D(Z)(D\ J=0)$Add new conditions to improve existing results.For example, we can combine the definition and Properties[8];9]To study the properties of Meromorphic solutions of complex linear differential equations,LaineAndWuIn the literature[10]In the case of homogeneous second order equation, the following results are obtained.

TheoremA[10]Design$A_0(Z)A_{1.}(Z)$Is an entire function and satisfies$(A_0)<(A_{1.})<\infty$And$T(R;_{1.})$ Log$M(R;_{1.})(R\to\infty;$ RWang Yi$E_{1.})$Where$E_{1.}$Meet$M_LE_{1.}<\infty$, Then Equation

$F''\ A_{1.}(Z)F'\ A_0(Z)F=0(2)$

$A_1)(R\to\infty;\ r\ E_2)$Which$E_2$MeetLog dens$E_2<((A_1)-(A_0))=(A_1)$Weak in conditions$T(R;_1)$

Log$M(R;_1)(R\to\infty;\ R/E_1)$Which$E_1$Meet$M_LE_1<\infty$.

Later,TuAndRongRespectively in Literature[14]And Literature[15]In TheoremBThe the promotion.

By the above results inspired we further study gap series in complex linear differential equation in the field of application.

First we will TheoremAAnd TheoremBIn the equation coefficients to meromorphic function situation and in iterative situation under get as follows Theorem1.

Theorem1SetD$\{0;1;$ In in; $k-1\};_J$(Twig U & Z); J= 0;1; In in; k-1; J =DAndF(Twig U & Z)For Meromorphic letter

Number,A_D(Twig U & Z)For the entire function and meet the following conditions

$\{\}$

$=\mathrm{Max}_P(A_J);_P(F)<_P(A_D)<\infty;$

$J=D$

$T(R;_D)\mathrm{Log}M(R;_D);R\to\infty;R\ \ E_2;$

WhichE_2MeetLog dens$E_2\underline{<}(_P(A_D)-)=_P(A_D)$.IfF(Twig U & Z)For Equation(1)Of Meromorphic solution and full

$_P(_F{}^1)<_P(F)$The$\leq_{P1}(F)\leq_P(A_D)$. Further to ifF(Twig U & Z)0TheF(Twig U & Z)Also meet

$\leq_{P1}(F)=_{P1}(F)=_{P1}(F)\leq_P(A_D)$:

Secondly we weakened TheoremAAnd TheoremBIn conditions willA_D(Twig U & Z)Of iterative level strict control role this a conditions weakenedA_D(Twig U & Z)Of iterative style the strict control role get as follows Theorem2.

Theorem2SetD$\{0;1;$ In in; $k-1\};_J$(Twig U & Z); J= 0;1; In in; k-1AndF(Twig U & Z)For the entire function and full

Foot the following conditions

$\mathrm{Max}_P(A_J);_P(F)\leq_P(A_D);$

$J=D$

$\mathrm{Max}_P(A_J):_P(A_J)=_P(A_D);\ J\neq D;_P(F)<_P(A_D);T(R;_D)$ Log$M(R;_D);\ R\to\infty;\ r\ \ E_1;$

WhichE_1Meet$M_LE_1<\infty$The equation(1)Of each beyond SolutionF(Twig U & Z)Meet$_{P1}(F)=_P(A_D)$. Further to ifF(Twig U & Z)0TheF(Twig U & Z)Also meet

$_{P1}(F)=_{P1}(F)=_{P1}(F)=_P(A_D)$:

Alignment times equation situation,HuangEt al in Literature[12]In also makes use of the limited deficit value conditions get the following results.

TheoremC[12]SetA_J(Twig U & Z); J= 0;1;In in $\cdots$; K-1AndF(Twig U & Z)(For0)For the entire function and meetMax$\{(A_J)$:

$\neq\ 0;\ D\}\ <(A_0)<(A_D)<\infty$.And set$A_0$(Twig U & Z)Meet$T(R;_0)$ Log$M(R;_0)(R\to\infty;\ r$ $E_1)$WhichE_1Meet$M_LE_1<\infty A_D$(Twig U & Z)Has limited deficit value the equation(1)Of each non-zero

Solution meet$(A_0)\leq_2(F)\leq(A_D)$.

We further will TheoremCIn the equation coefficients to meromorphic function situation get as follows Theorem3.

Theorem3SetA_J(Twig U & Z); J= 0;1;In in $\cdots$; K-1AndF(Twig U & Z)(For0)For Meromorphic Function,A_0(Twig U & Z)For the entire function and meetMax$\{(A_J):J\neq;0D\}<(A_0)<(A_D)<\infty$.And set$A_0$(Twig U & Z)Meet$T(R;_0)$ Log$M(R;_0)(R\to\infty;\ R/E_1)$Which$E_1$Meet$M_LE_1<\infty;_D$(Twig U & Z)Has limited deficit value.

F(Twig U & Z)For Equation(1)Of Meromorphic solution and meet$(_F{}^1)<(F)$The$(A_0)\leq_2(F)\leq(A_D)$.

Theorem1To Theorem3Promotion and improved the previous studies have been results rich and perfect the complex linear differential equation theory at the same time also rich the gap series in complex linear differential equation in the field of application. On the other hand Theorem1To Theorem3Also only is such problem of part many problems still remain to be found and solve worth further in-depth study.

2. Prove Theorem1To Theorem3Required of Lemma

I(D)<POrI(D) =PAnd$_P$(D) =<. And setTwig U & ZFor| Twig U & Z |=RThe meet| G(Twig U & Z)|=M(R; G)Of Point,$_G$(R)SaidG(Twig U & Z)The Center Index there is a logarithmic measure limited of collectionE(1;∞)Makes

| Twig U & Z |=R [0;1]EWhen have

WhichEIs a line measure limited of collection.

Lemma3[2]SetF(R)AndG(R)Is(0;∞)In non-decreasing function. IfF(R)≤G(R)Up to remove a line measure limited of exception set or whenR [0;1]HWhen,F(R)≤ G(R)WhichH(1;∞)Is a logarithmic measure limited of collection The for any given of constant>1ThereR_0>0Makes whenR> r_0When,

F(R)≤ G(R).

Lemma4[18]SetF(Twig U & Z)For beyond Meromorphic Function,>1For any to the constant the for any given">0Of

In:

ConstantB>0And a logarithmic measure limited of collectionE(1;∞)Makes of all meet| Twig U & Z |=R [0;1]EOfTwig U & ZHave

Line Measure zero of collectionH[0;2)And only rely on in the constantB>0Makes the any[0;2)\HThere exists a constantR_0=R_0()>0On all meetARGTwig U & Z=And| Twig U & Z |=R> R_0OfTwig U & ZHave

Lemma5[19]SetF(Twig U & Z)Meet Lemma1Of conditions there is a logarithmic measure limited of collectionE(1;∞)Makes whenTwig U & ZMeet| Twig U & Z |=R [0;1]EAnd| G(Twig U & Z)|=M(R; G)When have

Meromorphic solution and meet the following conditions one:

Max{I(F); I(A_J); J= 0;1;In in ···; K-1}<I(F) =P1;

Max{$_{P1}$(F);$_{P1}$(A_J); J= 0;1;In in ···; K-1}<$_{P1}$(F);

The

$_{P1}$(F) =$_{P1}$(F) =$_{P1}$(F):

Lemma10SetF(Twig U & Z)Is beyond entire function and meet0<$_P$(F)<∞;0<$_P$(F)<∞AndT(R; f) LogM(R; f)(R→ ∞; R/E$_1$)WhichE$_1$MeetM$_L$E$_1$<∞The for any given(<$_P$(F))There a logarithmic measure infinite of collectionE(1;∞)And line measure zero of collectionH[0;2), Making for all full

ProofYinM(R; f) =T(R; f) LogM(R; f)(R→∞$_{1.}$)WhereE$_{1.}$MeetM$_L$E$_{1.}$<∞So we assert that there is a set of zero line measuresH[0];2)To give">0And all

Otherwise, there is a set of line measures greater than zeroH_0[0];2)To give">0And all meet| Z |=

$^{2-"MH0}$LogM(R; f):2

">0; MH_0>0So on-andM(R; f) =T(R; f) LogM(R; f)(R → ∞; r E$_1$)Contradiction. To an assertion is established. The as to(<$_P$(F))Select Real Number$_1$Meet<(1-")$_1$<$_1$<$_P$(F). The by Lemma9The there a logarithmic measure infinite of collectionE_0(1;∞)Makes of all| Twig U & Z |=RE_0Have

Exp$_P${P$^{(F)}$}: M(R; f)$_1$R

RememberE=E_0\ E$_1$TheEWith infinite logarithmic Measure.So whenTwig U & ZMeet| Twig U & Z |=R e;ARGTwig U & Z=[0;2)\HWhen have

A constantK(;)And a collectionE(0;∞)WhichEMeetLog densE>1-Makes of allR EAnd has lengthLOfJHave

Lemma12[15]SetF(Twig U & Z)For beyond entire function and meet0<(F)<∞AndT(R; f)LogM(R; f)(R→

∞; R E$_1$)WhichE$_1$MeetM$_L$E$_1$<∞The for any given">0There collectionE(1;∞)And collectionH[0;2)Respectively meetLog densE>0AndMH= 0Makes of all meet| Twig U & Z |=R EAndARGTwig U & Z= [0;2)\HOfTwig U & ZHave

3. Theorem1To Theorem3Of prove

Theorem1.ProofDesignF(Z)For Equation(1)The meromorphic solution$_{P}(_{F}{}^{1.})<_{P}$(F)F(Z)Must be super

By Lemma1., Take the pointZMeet| Z |=RAnd| G(Z)|=M(R; G)There is a finite set of log measures$E_{3.}$(1);∞), Making when| Z |=RWang Yi[0];1]$E_{3.}$When there is

Will Equation(1)Rewrite

Select sufficiently smallL, MakingK((A_D);)(LLog$^{1.}{}_{L}$)<Thus, for all satisfaction| Z |=REAndARGZ

By Lemma4.It is known that there is a set of zero line measure$H_{3.}$[0];2)And constantD>0, Making for all full

| Z |=R→∞AndARGZ= [$_{R}$;$_{R}$L]\ H_3.OfZYes

Also knownT(R;$_0$)LogM(R;$_0$)(R→∞; RWang Yi$E_{1.}$)Where$E_{1.}$Meet$M_L E_{1.}$<∞,

= Log dens$E_{13.}$>0And take=$_{2.}$

Log dens (EU$E_{13.}$) = Log densELog dens$E_{13.}$-Log dens (E$E_{13.}$)

>1.--1 =$_{2.}$>0;

M_L(Eφe$_{13.}$) =∞. Will(29), (31)-(33)Substitution(30)For all satisfaction| Z |=R(EU$E_{13.}$)\($E_{1.}E_{14.}$)AndARGZ= [$_{R}$;$_{R}$L]\($H_{3.}H_{4.}$)OfZYes

$_{Exp}\{_{R}{}^{(A}0)''\}_{\leq KD}[_{T(2)R;\, f)}]^{2.K}{}_{Exp}\{_{R}{}''\}$;

(A_0)≤$_{2.}$(F).

Thus,(A_0)≤$_{2.}$(F)≤(A_D).

4. Conclusion

In this paper, the application of missing series in the field of complex linear differential equations is studied.NevanlinnaCombination of Theory

Fej' erThe definition and properties of a class of homogeneous and non-homogeneous Higher Order complex linear differential equations are studied.(1). When Equation(1)A factorA_D(Z)WithFej' erWhen the other coefficients are integral functions or meromorphic functions, the equations in the case of iteration are obtained.(1)The growth and value distribution of Meromorphic solutions(See Theorem1.Sum Theorem2)When the homogeneous equation(1)A factorA_0(Z)WithFej' erWhen the other coefficients are meromorphic functions, the equation is obtained.(1)Super estimate of Meromorphic solutions(See Theorem3). Theorem1.To Theorem3.This paper generalizes and improves the previous results, enriches and perfects the theory of complex linear differential equations, and also enriches the application of missing series in the field of complex linear differential equations.

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