# The Application of Combination of Number and Shape in High School Mathematics Teaching 

__Taking Function as an Example

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#### Abstract

Focusing on the idea of combination of number and shape, combining the latest research theories and classic examples in high school mathematics teaching, to explain in detail the specific use of the idea of combination of number and shape in function problems, so as to show that the idea of combination of number and shape is in the important value in high school mathematics learning. By consulting relevant research literature, the theory of the combination of number and shape is explained, and the classic examples of high school mathematics function are selected, and the method of using combination of number and shape is analyzed in detail. Finally, the applicable method of the combination of number and shape in different situations is obtained. At present, the idea of combining figures and shapes has strong practicability.


Keywords: combination of number and shape; high school mathematics; function

## 1.Introduction to the Concept of Combination of Number and Shape

In mathematics, the corresponding mathematical method is very important. The mathematical method is to use mathematical language to express the state, relationship and process of things, and to use derivation, calculation and analysis to form interpretation judgments and methods for the problem ${ }^{[1]}$. In the current high school mathematics, "number" and "shape" are two important roles, and their corresponding method is the combination of number and shape. The famous Chinese mathematician Mr. Hua Luogeng once said: "The combination of numbers and shapes is good, and everything is separated from each other." The concretization of things usually mainly includes two levels, namely, "use the shape to assist the number" and "use the number to solve the shape". Using this idea, we can solve problems such as sets, functions, sequences, and analytic geometry. As far as the function problem is concerned, it is mainly to use the combination of the function image and the function formula, through the image and its geometric meaning, so as to quickly obtain the hidden conditions.Therefore, "number" and "shape" complement each other and are indispensable.

## 2.Analysis of Educational Value of Combination of Number and Shape

Although the combination of number and shape is good in every possible way, what is its true value?

### 2.1Problem solving value

The value of problem-solving is one of the important values of the combination of number and shape, and it is also the value that can best be reflected in the process of using the combination of number and shape in our daily thinking. The subject of mathematics is originally an abstract subject. Most of the data, ideas, and topics it contains are
abstract things. As the difficulty of learning mathematics increases, some topics often contain hidden conditions, and these hidden conditions Special methods and thinking are often needed to dig, and these hidden conditions are often the key to solving problems ${ }^{[2]}$. The idea of combining numbers and shapes is a tool to help us dig out hidden conditions. Looking at the essence through the problem, our most fundamental purpose is to make the complicated problem at hand concise and clear. Maybe some mathematical problems can be solved without using the idea of combining numbers and shapes, but the process will be more cumbersome, which is often error-prone, so it is reasonable. The idea of combining numbers and shapes can effectively simplify the problem. When facing some difficult math problems and there is no idea to solve the problem, try to use the combination of number and shape, maybe he will point you to the direction of solving the problem. Although what geometric figures show us is concrete and intuitive, in terms of accuracy, we still need "number" to help. The accuracy of "number" coupled with the intuitive specificity of geometric figures can help us better go Solve the problem.

### 2.2 Thinking training value

In the subject of mathematics, intuitive thinking occupies a large part. When solving a mathematical problem, students use the mathematical knowledge they have learned to make quick judgments on the objects and structure of mathematics, so as to make assumptions and draw conclusions. In conclusion, it is characterized by a leap-forward way of thinking ${ }^{[3]}$. Mathematics has the two characteristics of rigorous and flexible logical thinking, which requires our problem-solving thinking to be flexible and rigorous. We need to look at and solve problems from different angles. In the daily problem-solving process, getting answers is not the ultimate goal. The goal is to get the optimal solution among all problem-solving methods and finally solve the problem efficiently. The human brain is divided into the left hemisphere and the right hemisphere. The left hemisphere is in charge of the calculation and reasoning of abstract data, and the right hemisphere is in charge of thinking about visual and intuitive things. We must be good at converting text and digital information into image information, replacing abstract thinking with image thinking, and then using image thinking to in turn promote the cultivation of abstract thinking, and at the same time let us develop an intuitive thinking ability. In the process of solving mathematics problems, the first feeling of the problem, that is, intuition is very important. The combination of number and shape is to let our left hemisphere and right hemisphere operate at the same time, and the two work together to solve problems together.

### 2.3 Other value

The combination of number and shape plays a very important role in solving mathematical problems, and it also has a certain effect on the improvement of our brain memory. In daily life, we need to spend a lot of time to recite and remember those abstract words and algebras, but for those intuitive and concrete images, we can easily remember them ${ }^{[4]}$, so we might as well Converting these abstract algebraic predictions into concrete graphic symbolic languages, then we can more easily remember these graphic symbolic languages to replace those obscure algebraic languages.

## 3.The application of combination of number and shape in the process of solving high school mathematics function problems

Among the many practical values of the combination of number and shape, the most important is his problemsolving value, so we need to show his problem-solving value through specific classic examples.

### 3.1Three types of thinking methods combining number and shape

The idea of combining number and shape mainly includes three ideas: "Shaping number", by analyzing the quantitative relationship in the figure, transforming a geometric problem into an algebraic problem, and then solving the problem through calculations. "Numerical form", to solve the problem by turning complex algebraic problems into simpler geometric problems and digging out the hidden conditions in the problem. "Number and shape conversion"
is usually used to solve the problem of abstract functions, which requires mutual transformation between graphics and algebra, and the use of comprehensive thinking to solve problems.

### 3.2Shaped number-functional equation problem

In the function problem related to the method of "shaping the number", it is usually known that all or part of the function image and function form are known, and then the corresponding function expression is required to be derived from the function image ${ }^{[5]}$. In this type of problem, The most typical is the problem of trigonometric functions. To solve this kind of problem, we usually need to know the image properties related to the desired type function first, and then crack the unknown constant terms in the expression one by one.

Example:Given the function

$$
\begin{equation*}
\mathrm{f}(x)=A \sin (\omega x+\varphi) \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\left(A>0,|\varphi|<\frac{\pi}{2}, \omega>0\right) \tag{2}
\end{equation*}
$$

part of the image is shown in the figure, find the expression of $\mathrm{f}(x)$ and its symmetry axis equation.


Solution:According to the given function image and the properties of the trigonometric function, we can get that the maximum value of the function $f(x)_{\max }=2$, so we can get $A=2$, and the point $(0,1)$ is on the function image, bring this point into the function Expression

$$
\begin{equation*}
f(x)=2 \sin (\omega x+\varphi) \tag{3}
\end{equation*}
$$

you can get

$$
\begin{equation*}
1=2 \sin (\omega \times 0+\varphi) \tag{4}
\end{equation*}
$$

after calculation $\varphi=\frac{\pi}{6}$; then we find that the point $\frac{11 \pi}{12}, 0$ is also on the function image, so we can get

$$
\begin{equation*}
2 \sin \left(\frac{11 \pi}{12} \times \omega+\frac{\pi}{6}\right)=0 \tag{5}
\end{equation*}
$$

after calculation $\omega=2$, so the expression of the original function equation is

$$
\begin{equation*}
f(x)=2 \sin \left(2 x+\frac{\pi}{6}\right) \tag{6}
\end{equation*}
$$

Then we find the symmetry axis equation of the function. First, we first draw the image of the function
$g(x)=\sin x$,


We can get the symmetry axis equation of the function $g(x)$ as

$$
\begin{equation*}
x=k \pi+\frac{\pi}{2}(k \in Z) \tag{7}
\end{equation*}
$$

then if it is to find the symmetry axis of $f(x)$, we can regard $\left(2 x+\frac{\pi}{6}\right)$ as a whole and get

$$
\begin{equation*}
2 x+\frac{\pi}{6}=k+\frac{\pi}{2} \tag{8}
\end{equation*}
$$

so the symmetry axis equation of the equation $f(x)$ is

$$
\begin{equation*}
x=\frac{k \pi}{2}+\frac{\pi}{6}(k \in Z) \tag{9}
\end{equation*}
$$

and the points can be obtained according to the $f(x)$ function image $\left(\frac{\pi}{6}, 2\right)$ is on the symmetry axis of the function $f(x)$ at the same time, and $k=0$ can be obtained by bringing it in, so the symmetry axis equation is established.

Comment: According to the function image, looking for intuitive and useful information can often quickly get some useful data, avoiding tedious calculations.

### 3.3Digital shape-the problem of function monotonicity

In the function problem related to the method of "digitalization", usually the title informs the relevant function expression, and then calculates the monotonic interval, monotonicity, increase and decrease of the function based on the expression ${ }^{[6]}$. Taking the problem of finding the monotonicity and monotonic interval of a function as an example, we can use traditional calculations to find the maximum, minimum, and symmetry axis according to the nature of the function to determine the monotonicity and monotonic interval of the function. However, in this type of problem, the calculation is often too complicated and easy to make mistakes, but if we use the idea of combining numbers and shapes and combine the corresponding function images, we can intuitively see the monotonic interval and monotonicity of the function.

Example: Find the monotonic interval of function

$$
\begin{equation*}
f(x)=x|x|-3|x| \tag{10}
\end{equation*}
$$

Solution:It is not difficult to see that this function is a piecewise function, so we can expand it first to get

$$
f(x)=\left\{\begin{array}{l}
x^{2}-3 x, x>0  \tag{11}\\
-x^{2}+3 x, x<0
\end{array}\right.
$$

and then draw the original function image according to these two one-dimensional quadratic equations:


So we can get the monotonic interval of the function $f(x)$ is when the range of x is $(-\infty, 0) \cup(1,+\infty)$, the function increases monotonously, and when the range of $x$ is $(0,1)$, the function monotonously decreases.

Comment: If the function expression given in the title is a general function that we are familiar with, then we can directly make his image and get the answer easily and quickly, avoiding tedious and error-prone calculations ${ }^{[7]}$.
3.4Number and shape conversion-abstract function problem

The idea of "number-shape conversion" is actually a combination of the previous two methods. In the face of some more complex abstract function problems, we can comprehensively and flexibly use the mutual conversion between "number" and "shape" to achieve simplification.

Example: If the image of the quadratic function $y=f(x)$ passes through the origin, and

$$
\begin{equation*}
1 \leq f(-1) \leq 2,3 \leq f(1) \leq 4 \tag{12}
\end{equation*}
$$

find the value range of $f(-2)$.
Solution: It is not difficult to see that this topic does not give us a clear function expression, but only tells us that this is a quadratic function, so this is an unknown abstract function, so we need to set the original function expression

$$
\begin{equation*}
y=f(x)=a x^{2}+b x \tag{13}
\end{equation*}
$$

according to limited conditions, So get

$$
\left\{\begin{array}{l}
1 \leq a-b \leq 2  \tag{14}\\
3 \leq a+b \leq 4
\end{array}\right.
$$

according to

$$
\left\{\begin{array}{l}
1 \leq f(-1) \leq 2  \tag{15}\\
3 \leq f(1) \leq 4
\end{array}\right.
$$

and then make the image of this set of inequalities:


The intersecting part in the figure is the range represented by the unequal group, because

$$
\begin{equation*}
f(-2)=4 a-2 b \tag{16}
\end{equation*}
$$

so when

$$
\begin{equation*}
4 a-2 b-f(-2)=0 \tag{17}
\end{equation*}
$$

and passing the point

$$
\begin{equation*}
A(2,1) \quad B(3,1) \tag{18}
\end{equation*}
$$

take the

$$
\begin{equation*}
f(-2)_{\min }=6, f(-2)_{\max }=10 \tag{19}
\end{equation*}
$$

so the solution of this problem is

$$
\begin{equation*}
f(-2) \in(6,10) \tag{20}
\end{equation*}
$$

Comment: For the problem of abstract function classes, we have to sort out the known conditions, narrow the range of function types, list function expressions with unknown parameters as much as possible, and then still make images based on existing conditions, and find new ones in the images the implied condition ${ }^{[8]}$.

## 4.Summary

Engels once said: "The main function of mathematics is to explore the spatial form and quantitative relationship of the world" ${ }^{\prime 9]}$. Number and form are the two cornerstones in the process of mathematics learning, and the two merge and penetrate each other.The combination of number and shape can convert graphic information into algebraic information, or construct geometric figures based on the structural characteristics of the quantity ${ }^{[10]}$. In the study of high school mathematics, we must be good at divergent thinking and explore ways to solve problems. Although no matter which method is correct, we can get the answer, but we should raise a level of thinking and we have to find out The optimal solution is not a simple positive solution. The idea of combining figures and shapes is a practical idea that helps us to open up our thinking when solving problems. The method of combining numbers and shapes is intuitive and flexible. It can help us remember the topic more deeply. In the final analysis, the "combination of number and shape" breaks the "barrier" between various knowledge points, and in a real sense connects the knowledge points, allowing us to get the final answer faster.

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